NETWORK ANALYSIS [EET251]

BE III SEMESTER-EE

CHAPTER – 01

LECTURE-01

COMMON TERMS IN ELECTRICAL NETWORKS

DATE: 06-07-2020

12.00 – 12.45 AM

TEACHER'S NAME:
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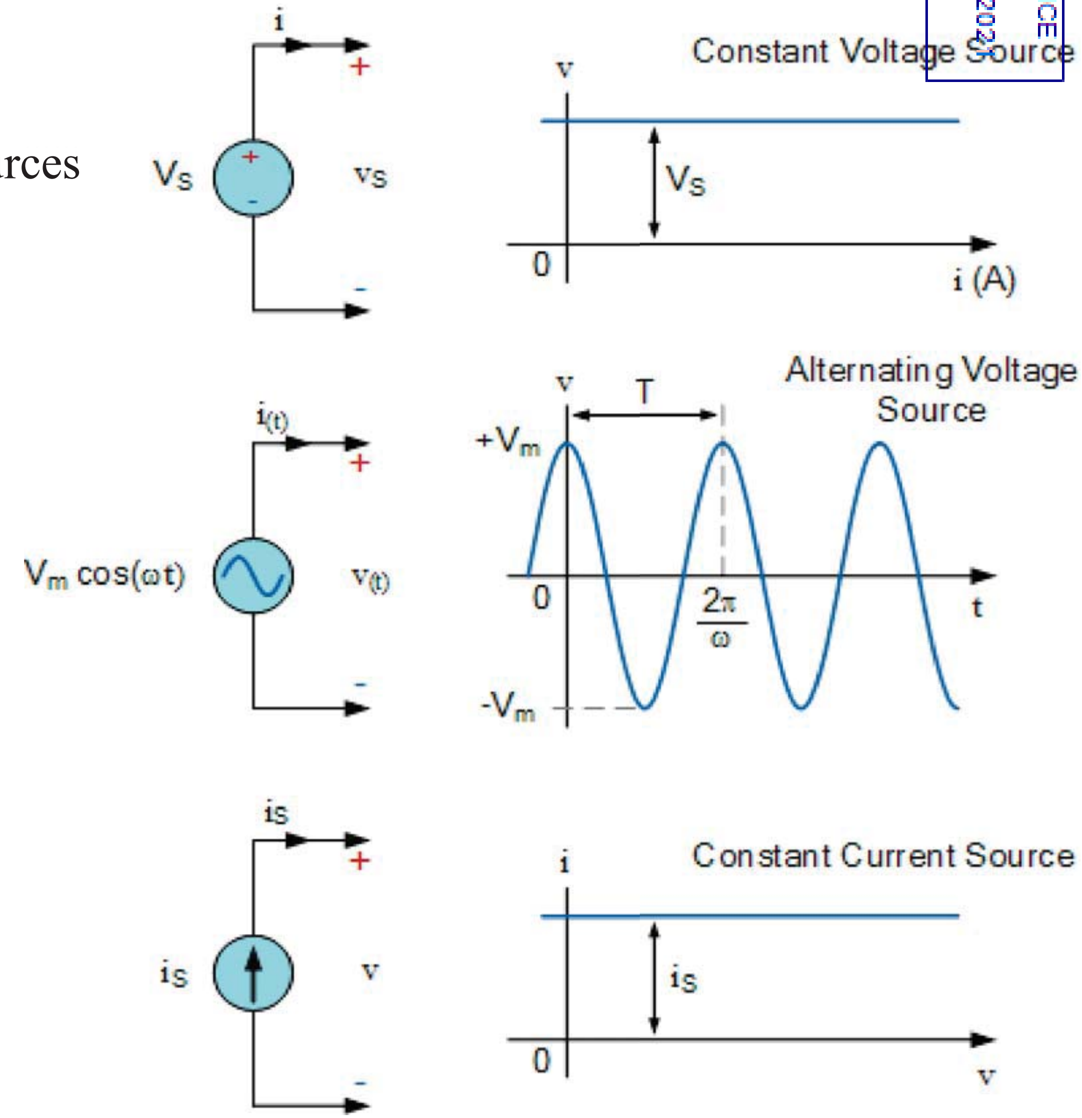
Module – 1.1

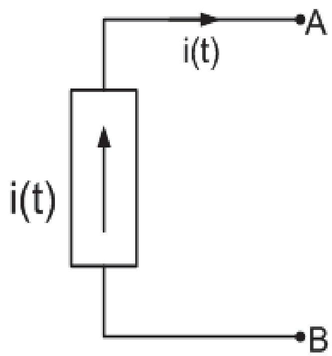
ELECTRICAL SOURCES

Classification

1. AC or DC Sources
2. Voltage or Current sources
3. Independent or Dependent Sources
4. Ideal or Practical Sources



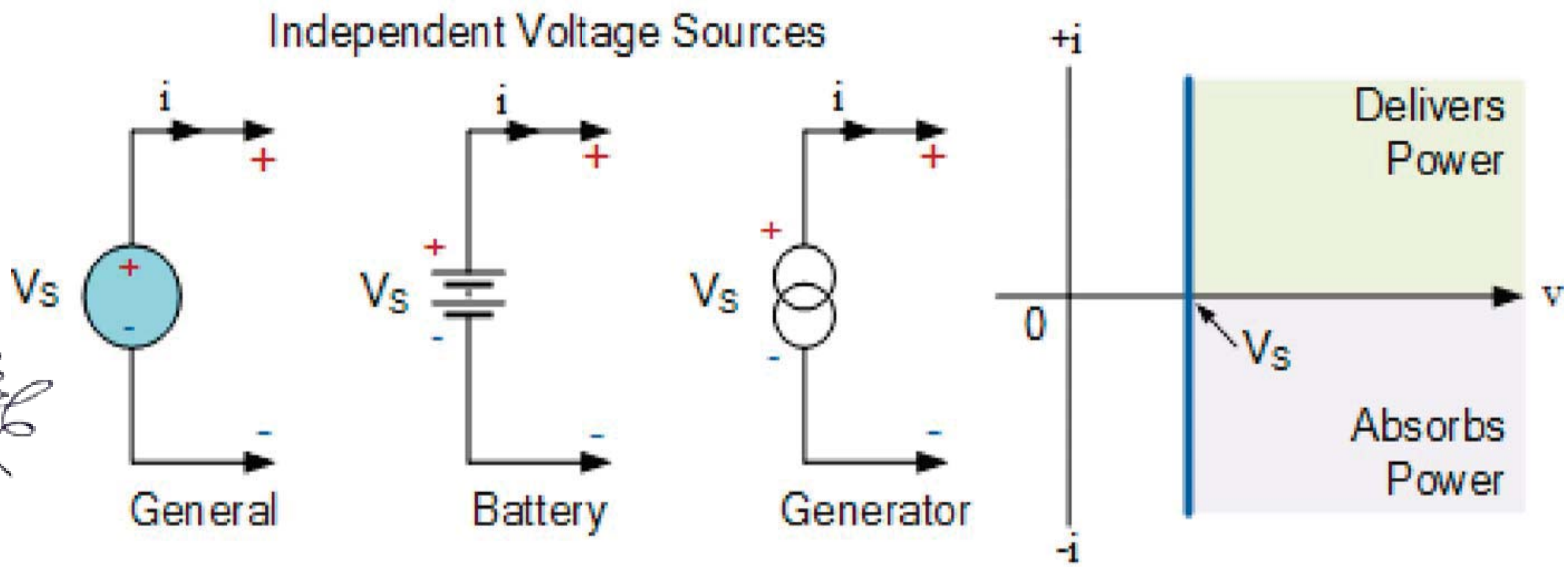
General representation of
Voltage and Current sources→



Independent Sources

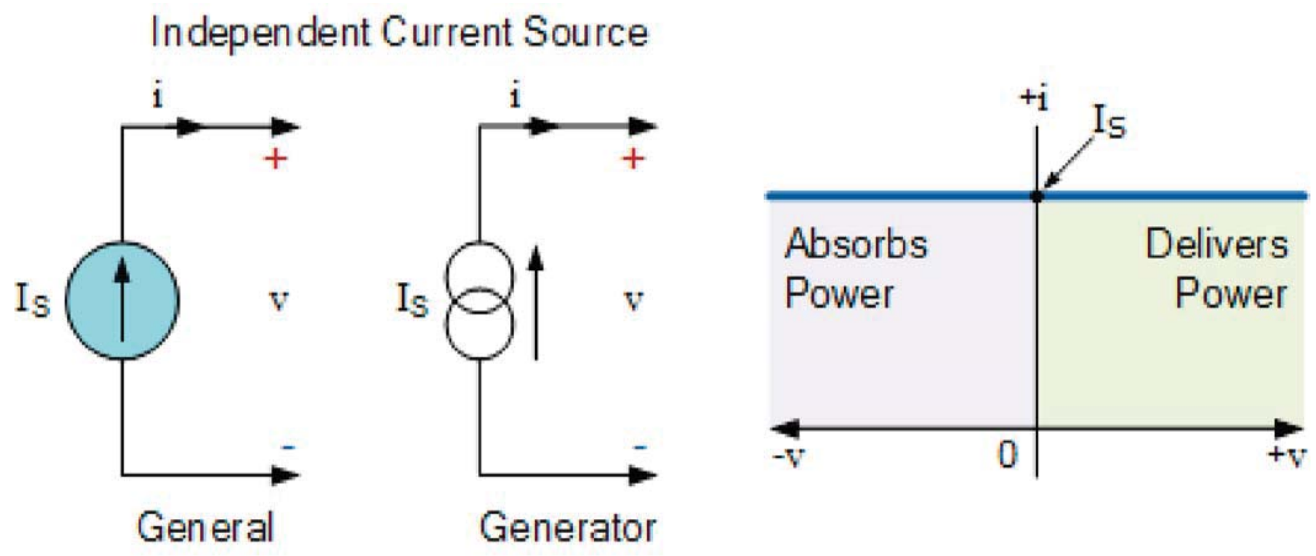
- Independent Voltage Sources

  The ideal voltage source is known as an **Independent Voltage Source** as its voltage does not depend on either the value of the current flowing through the source or its direction but is determined solely by the value of the source alone.



Independent Current Sources

Ideal Current sources: The ideal current source is called a "constant current source" as it provides a constant steady state current independent of the load connected to it producing an I-V characteristic represented by a straight line



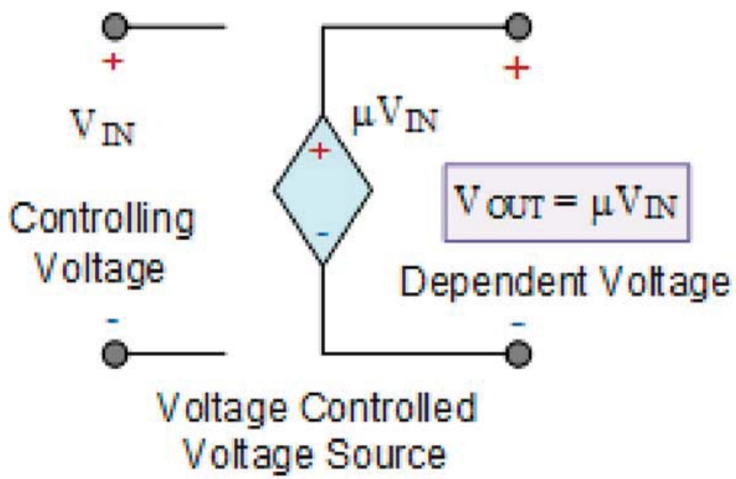
Dependent Source

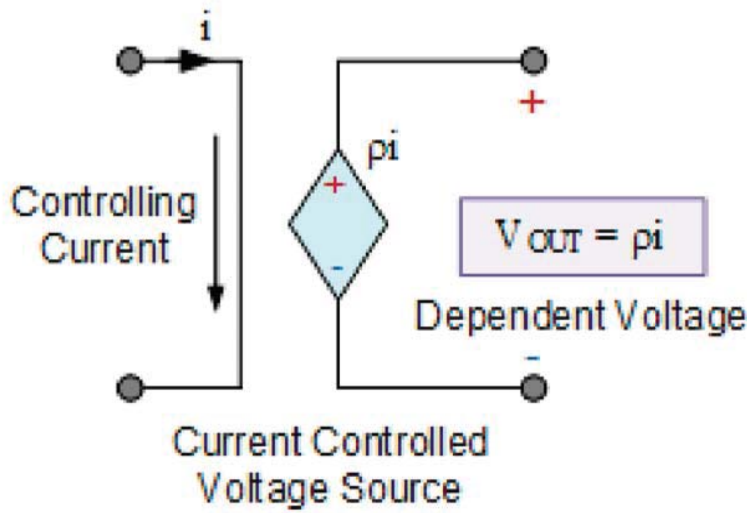
- Dependent Voltage Source

A controlled or dependent source changes its terminal voltage/current depending upon the voltage across, or the current through, some other element connected to the circuit.

They could be classified as,

1. Voltage controlled voltage sources or
2. Current controlled voltage sources



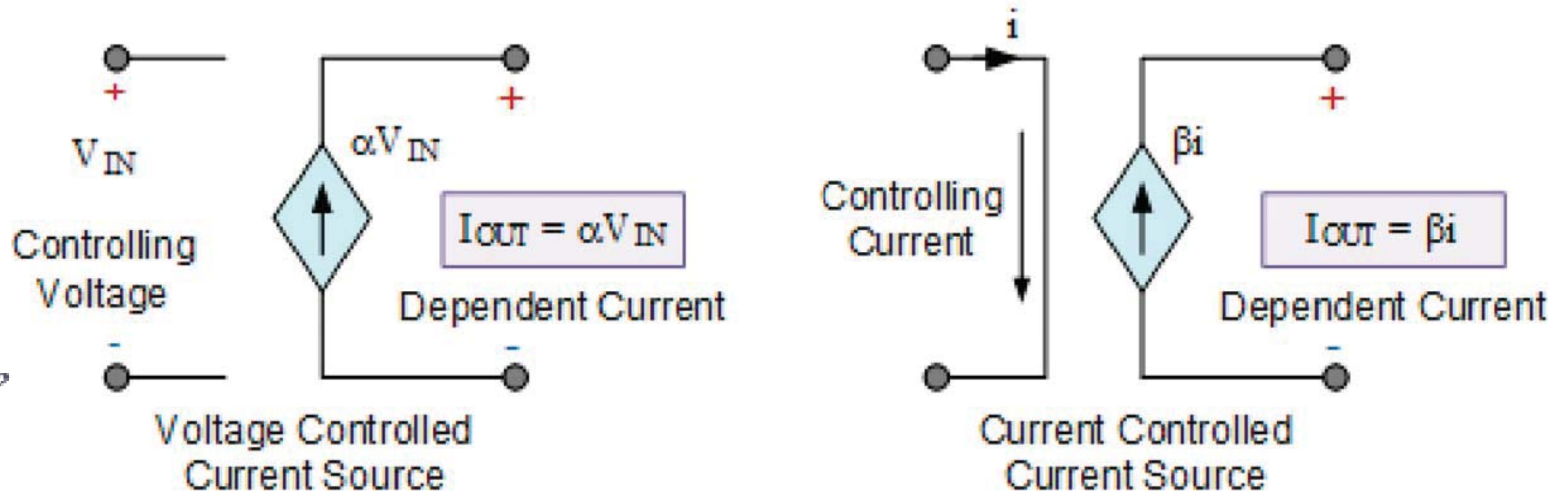


Dependent Current Sources:

An ideal dependent source, maintains an output current that is proportional to a controlling input current. Then the output current "depends" on the value of the input current, again making it a dependent current source.

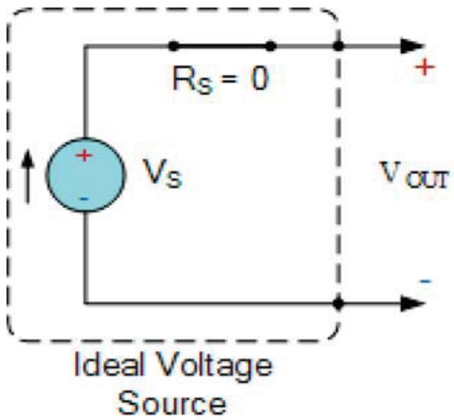
They also could be classified as,

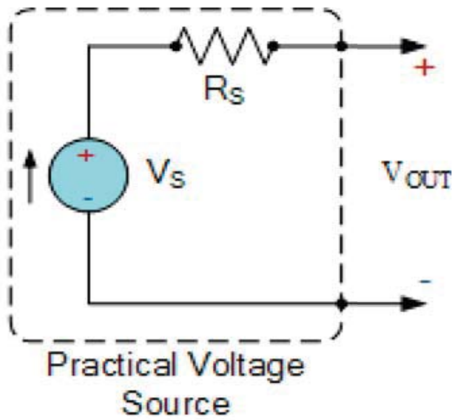
1. Voltage controlled current sources or, VCCS
2. Current-controlled current source, CCCS



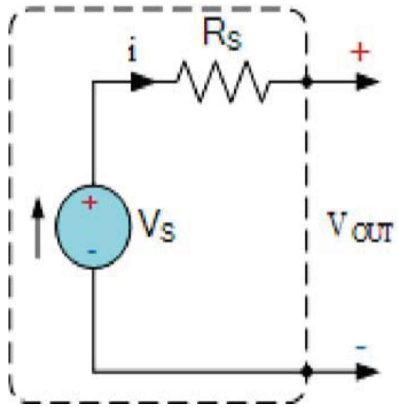
Ideal And Practical Sources

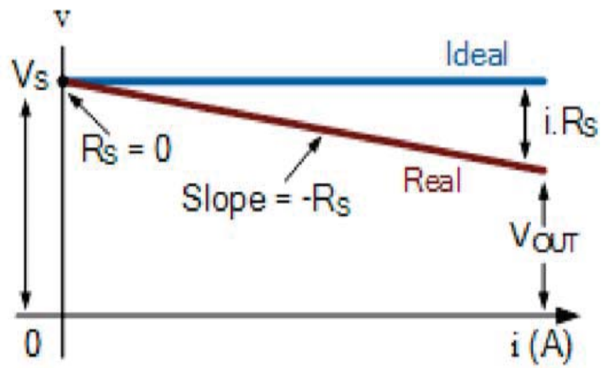
❖Voltage Source



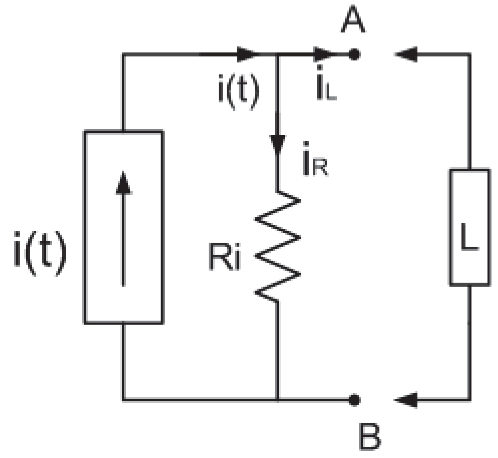


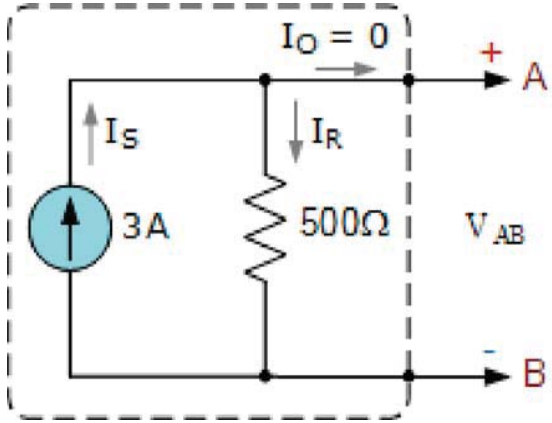
❖Practical Voltage Source





Practical Current Source





$I_S = I_R = 3A, \quad R_P = 500\Omega$

$V_{AB} = V_{RP} = I_S \times R_P$

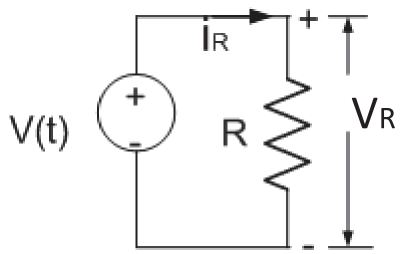
$\therefore V_{RP} = 3 \times 500 = 1500V$ or $1.5kV$

$P_R = I_S^2 \times R_P$

$P_R = 3^2 \times 500 = 4500W$ or $4.5kW$

Important Electrical Elements (Components)

Resistance



In resistive circuit, voltage is strictly proportional to the current, and this proportionality defines the resistance.

i.e. $R = V/I$

ductance:

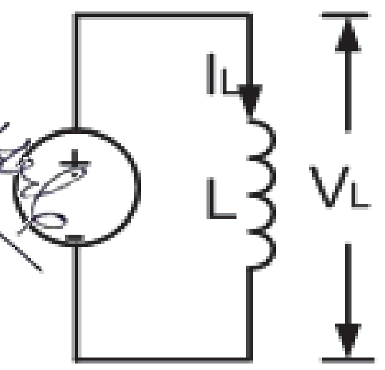
In inductive circuit, voltage is not strictly proportional to the current, but it varies with the differentiation of current w.r.t. time i.e.

- E_L or
$$V_L = L\frac{di_L}{dt}$$

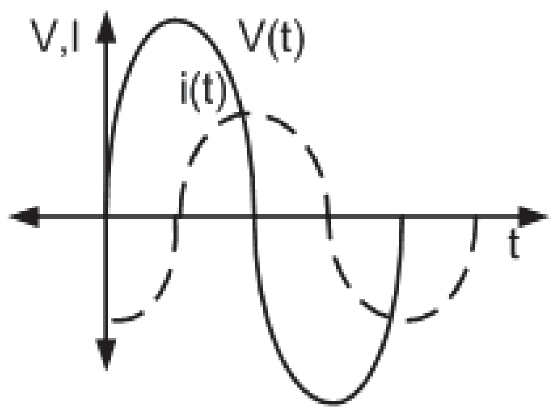
  As, by the Faraday's law of electromagnetic induction,

  e_L is,
$$e_L = -N\frac{d\emptyset}{dt} = \frac{d\lambda}{dt} = \frac{d\lambda}{di} \times \frac{di}{dt} = L\frac{di}{dt}$$

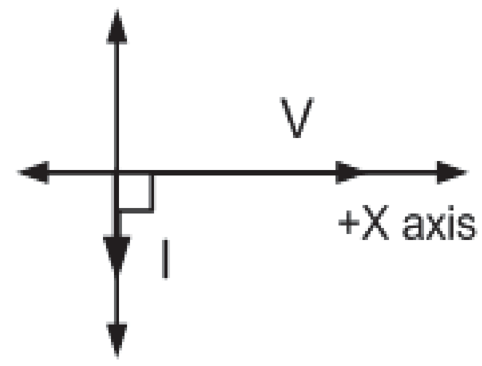
- Inductor can store the enrgy in the form of magnetic field, $W = \frac{1}{2}Li^2$ Watts



Inductive Circuit



AC voltage & Current



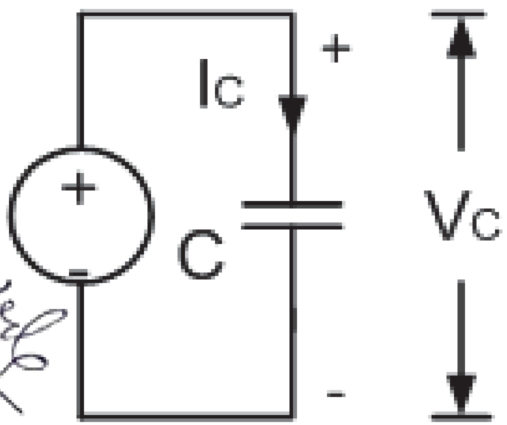
Voltage & Current Phasors

apacitance

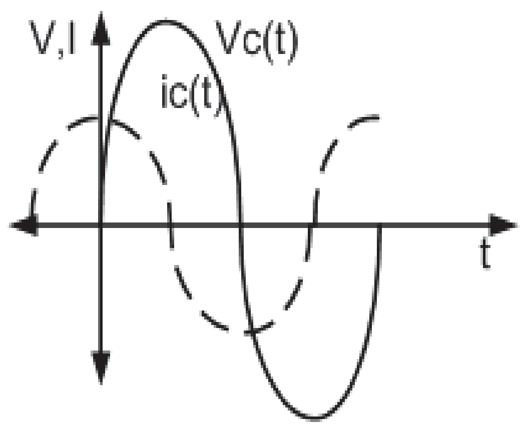
In inductive circuit, current is not strictly proportional to the voltage, but it varies with the integration of voltage w.r.t. time. The relationship for capacitor we know is, q(t)=Cv(t). Hence,

$$i_c = \frac{dq}{dt} = C\frac{dv(t)}{di} \quad \text{Or} \quad Vc(t) = \frac{1}{C}\int i_{c(t)}\,dt$$

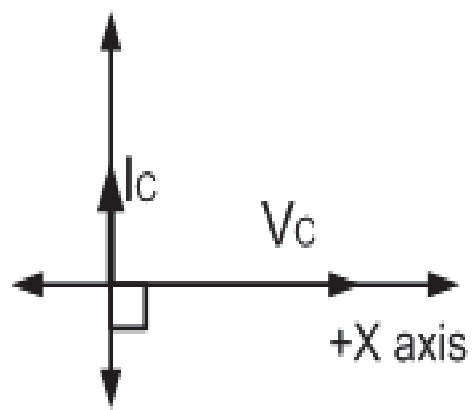
- Capacitor can store the energy in the form of magnetic field, $W = \frac{1}{2}Cv^2$ Watts.



Capacitive Circuit



AC voltage & Current

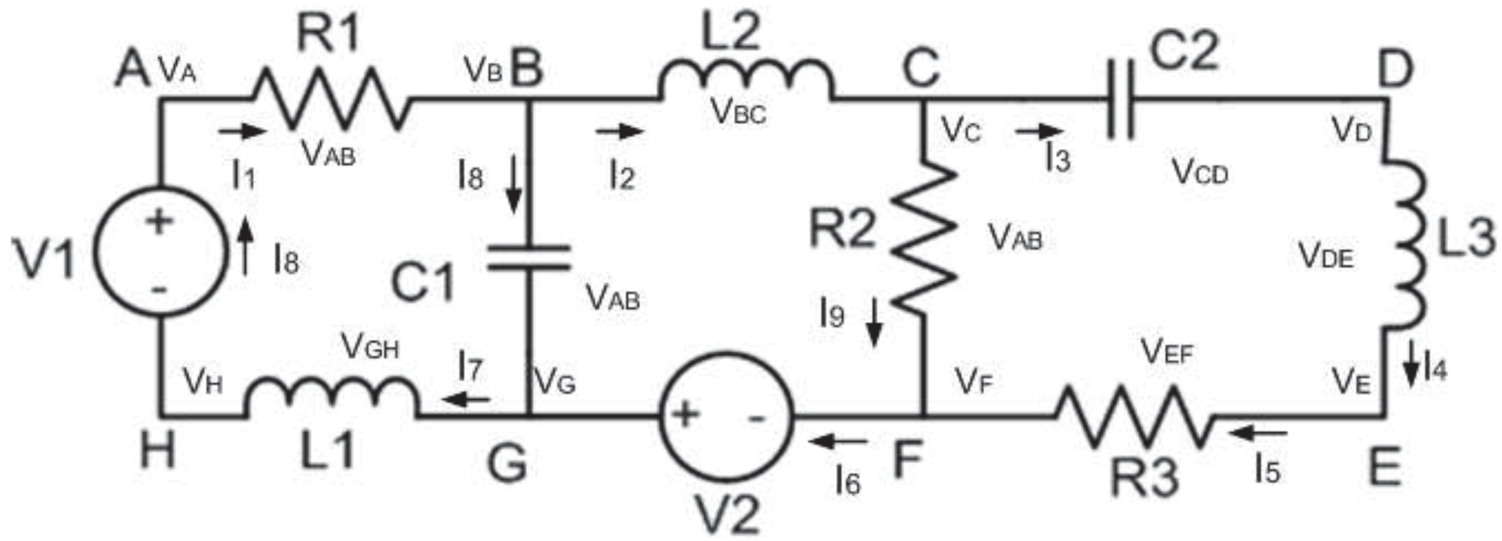


Voltage & Current Phasors

Module – 1.2

EQUILIBRIUM EQUATIONS

- The meaning of equilibrium (Law of conservation of energy)
- Importance of Equilibrium Equations
- Need to evaluate for each element,
  1. The current through,
  2. The voltage across and
  3. The power consumed
- If ‘e’ are total elements, ‘3e’ will be total quantities to evaluate
- Inter-related terms. One can be evaluated from the knowledge of other.
- Not all voltages and currents are independent of each other.



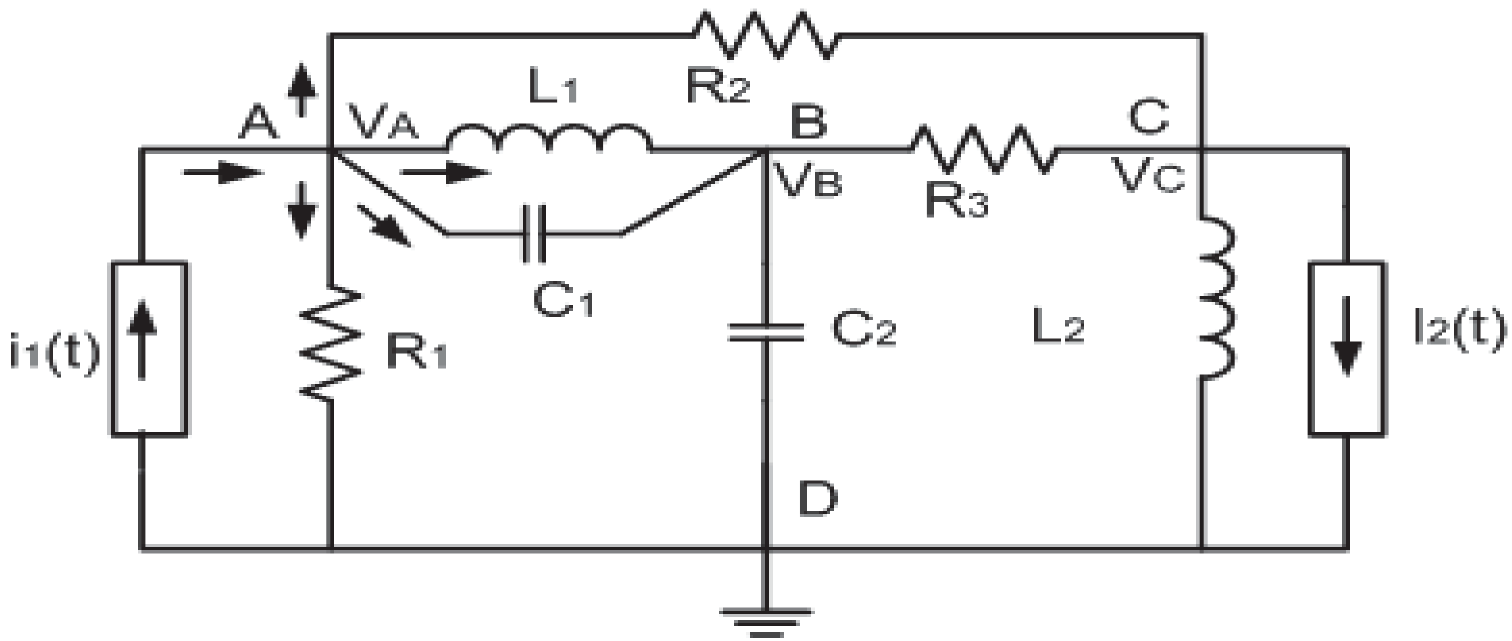
Some currents / voltages are always dependent on other currents / voltages in the circuit.

- So, there is always a set of independent currents/voltages .
- We have to fix the minimum number of these currents / voltages forming this set.
- It is always sufficient to find these independent currents /voltages of a problem.
- We can not miss the finding of a single quantity of these independent quantities.
- We shall first consider a case of finding the set of independent voltages of a given circuit.
- These are the voltages at every node, called **node voltages**.
- One node out of the total nodes, is to be considered as the reference or **datum node**.
- Voltage et datum node is always zero.
- Other nodes are called as **real (Independent) nodes** , in number, say "n" (Considering total nodes "n+1").
- The analysis used to find these node voltages is called as "**NODAL ANALYSIS**".

NODAL ANALYSIS

There are **four nodes** in the ckt given below, A, B, C and D. "n+1 = 4, so, n=3.

- We have to solve for 3 node voltages, V_A, V_B,& V_C. Node D is datum, with zero voltage
- In nodal analysis, we apply the KCL at each real node and write the equilibrium question.
- We assume that, *the node at which we stand and apply KCL is at* ***higher potential*** *than all other.*
- We again assume that, *the currents* ***entering the node are –ve*** *and those* ***leaving the node are +ve .***



First we shall write equation for node “A”.

There at total 5 currents, one entering and 4 leaving from node “A”.

- First entering current , by current source = $-\dot{i}_1(t)$ ***(-ve as entering)***
- Second leaving current, through resistor $R_1 = V_A/R_1$.
- Third leaving current, through the inductor L_1 connected between A & B =

$$\frac{1}{L_1}\int_{-\infty}^{t}(V_A - V_B)dt$$

- Fourth leaving current through capacitor C1connected between A & B =

$$C_1\frac{d(V_A - V_B)}{dt}$$

- Fifth leaving current passing through a resistor connected between A & C =

$$\frac{(V_A - V_c)}{R_2}$$

- Now, we write them together, using KCL,

$$-\dot{i}_1(t) + V_A/R_1 + \frac{1}{L_1}\int_{-\infty}^{t}(V_A - V_B)dt + C_1\frac{d(V_A - V_B)}{dt} + \frac{(V_A - V_c)}{R_2} = 0$$

- We shall simplify this equation , for making it more handy and useful.
- The Equilibrium should be written in terms of all it’s unknown veriables, here V_A, V_B & V_C.

The current through Inductor:->

$$i_{L1} = \frac{1}{L_1}\int_{-\infty}^{t}(V_A - V_B)\,dt = \frac{1}{L_1}\int_{-\infty}^{0}(V_A - V_B)\,dt + \frac{1}{L_1}\int_{0}^{t}(V_A - V_B)\,dt$$

❖ The integration could **be broken into two terms**.
The first term is integration with **limits -∞ to 0** and second with **limits 0 to t.**
The first term, represents the current **before** we start the analysis, called as **prior history**
Prior history is always useful to take into account which results in correct answer.
But it must be given in the problem. Here we assume that, it is **not present & not given**.
Only second term will remain here which is a regular term with limits 0 to t.

$$\frac{1}{L_1}\int_{-\infty}^{t}(V_A - V_B)\,dt = \frac{1}{L_1}\int_{0}^{t}(V_A - V_B)\,dt$$

❖ The current through capacitors will be same as shown, i.e. $i_{C1} = C_1 \frac{d(V_A - V_B)}{dt}$
We shall use now the “p operator”, which makes the equation more handy and re-writable.
So, we shall replace $\frac{d}{dt}$ by p & we write, $i_{C1} = C_1\, p(V_A - V_B)$
Where C_1 p is called as the **operational admittance** of the capacitor C_1.

❖ We are using here KCL and here all terms of equilibrium equation, are currents.
So for any element or circuit, admittance x voltage across it is the current i.e. I=Y x V .
In the same way, the **operational admittance** of the inductance will be $\frac{1}{pL_1}$. We are using $\frac{1}{p}$,but not p here, as its an integration is used for the inductance.

Now, the simplified equation looks like this,

), the eq is

$$\frac{1}{L_1}\int_{-\infty}^{t}(V_A - V_B)\,dt + C_1\frac{d}{dt}(V_A - V_B) + \frac{V_A}{R_1} + \frac{(V_A - V_C)}{R_2} - i_1(t) = 0$$

- We can shift the current term to the right side, collect all the terms of same variables (i.e. V_A, V_B, V_C) & rewrite the equation to get most simplified equation,

$$\left(\frac{1}{R_1} + \frac{1}{L_1 p} + C_1 P + \frac{1}{R_2}\right)V_A + \left(-\frac{1}{L_1 P} - C_1 P\right)V_B + \left(-\frac{1}{R_2}\right)V_C = i_1(t) \quad \text{----------(1)}$$

- In the same way we can write the equation at node " B".

$$\frac{1}{L_1}\int_{-\infty}^{t}(V_B - V_A)\,dt + C_1\frac{d}{dt}(V_B - V_A) + C_2\frac{d}{dt}V_B + \frac{(V_B - V_C)}{R_3} = 0$$

- Here also we have applied the KCL to node "B", in its original form. Here we assumed the voltage V_B is greater than all other voltages.
- At this node there is no any current source, hence equal to zero.

$$\frac{1}{L_1}\int_{-\infty}^{t}(V_B - V_A)\,dt + C_1\frac{d}{dt}(V_B - V_A) + C_2\frac{d}{dt}V_B + \frac{(V_B - V_C)}{R_3} = 0$$

- We apply again same rules and can write the simplified eq as,

$$\frac{I}{L_1 p}(V_B - V_A) + C_1\,p(V_B - V_A) + C_2\,p(V_A - V_B) + \frac{(V_A - V_C)}{R_2} = 0$$

And hence the final simplified eq is

$$\frac{V_A}{R_1} + \frac{I}{L_1 p}(V_A - V_B) + C_1\,p(V_A - V_B) + \frac{(V_A - V_B)}{R_2} - i_1(t) = 0 \quad \text{-------------(2)}$$

$$\left(-\frac{I}{L_1 p} - C_1\,p\right)V_A + \left(C_1\,p + \frac{I}{L_1 p} + C_2\,p + \frac{1}{R_3}\right)V_B + \left(-\frac{1}{R_3}\right)V_C = 0$$

- Now we write the equilibrium eq for node "C" .

$$\frac{(V_C - V_B)}{R_3} + \frac{(V_C - V_A)}{R_2} + \frac{1}{L_2}\int_0^t V_C\, dt + i_2(t) = 0$$

- And the simplified eq for node C is,
- $\left(-\frac{1}{R_2}\right)V_A + \left(-\frac{1}{R_3}\right)V_B + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 p}\right)V_C = -i_2(t)$ -------------(3)
- Thus the final three eqs are,
-

$$\left(\frac{1}{R_1} + \frac{1}{L_1 p} + C_1 P + \frac{1}{R_2}\right)V_A + \left(-\frac{1}{L_1 P} - C_1 P\right)V_B + \left(-\frac{1}{R_2}\right)V_C = i_1(t)$$ -------------(1) -------------(2)

$$\left(-\frac{1}{R_2}\right)V_A + \left(-\frac{1}{R_3}\right)V_B + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 p}\right)V_C = -i_2(t)$$ ------------(3)

We can write them in matrix form

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{L_1 p} + C_1 P + \frac{1}{R_2}\right) & \left(-\frac{1}{L_1 P} - C_1 P\right) & \left(-\frac{1}{R_2}\right) \\ \left(-\frac{I}{L_1 p} - C_1 p\right) & \left(C_1 p + \frac{I}{L_1 p} + C_2 p + \frac{1}{R_3}\right) & \left(-\frac{1}{R_3}\right) \\ \left(-\frac{1}{R_2}\right) & \left(-\frac{1}{R_3}\right) & \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2 p}\right) \end{bmatrix} \times \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_1(t) \\ 0 \\ -i_2(t) \end{bmatrix}$$

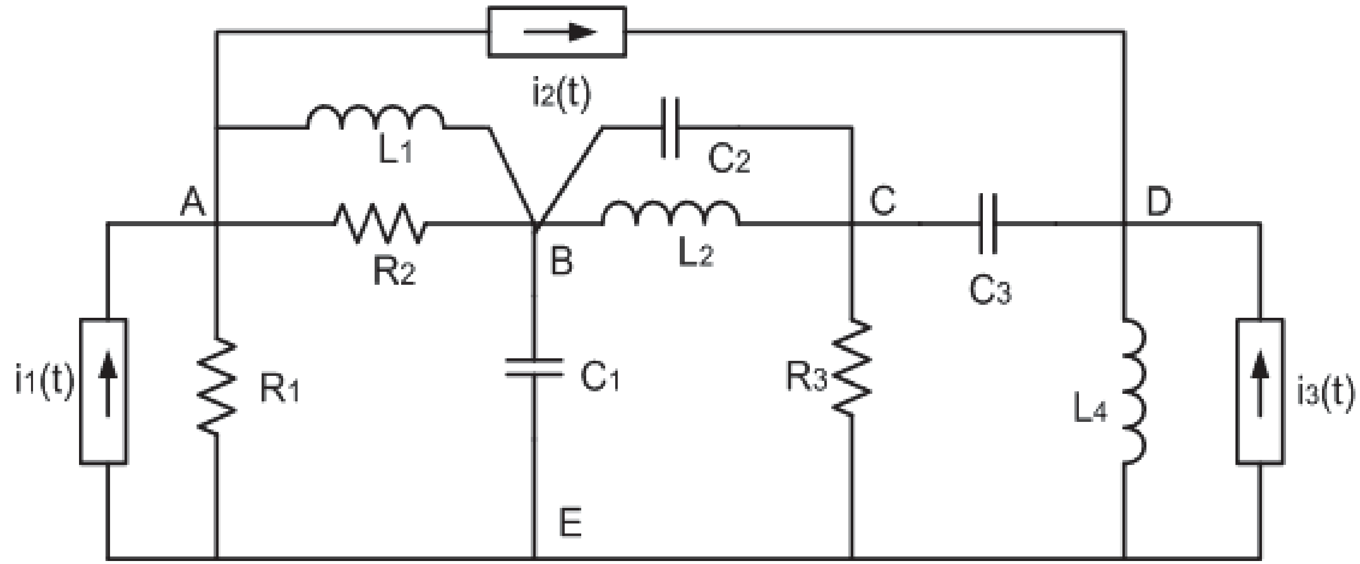
In this 3 x 3 matrix all entries are admittances and it's a square matrix, which is called as the **Nodal Admittance Matrix.**

- So, it follows the rule that, $[Y] \times [V] = [I]$
- $[Y]$ is nxn matrix, $[V]$ is nx1 i.e. column matrix and $[I]$ also is nx1 i.e. column matrix.
- $[Y]$ is **Nodal Admittance Matrix** could be shown as,

$$[Y] = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

- The entries with same suffices i.e. y_{11}, y_{22} and y_{33} are called as the **diagonal entries**.
- And the entries with different suffices i.e. y_{12}, y_{13}, ..y_{32} are called as **off- diagonal entries**.
- The diagonal entries, 'y_{kk}' are the positive sum of the admittances of all the branches meeting at "A", they are called as the **self -admittances** of node A.
- The off-diagonal entries 'y_{jk}' are, the negative sums of admittances of the branches connected between the node in hand "A" and all other neighboring nodes. These admittances are called as the **mutual admittances.**
- The off-diagonal entries 'y_{jk}' follow the rule that $y_{jk} = y_{kj}$ i.e. $y_{12} = y_{21}$, $y_{23} = y_{32}$ etc..
- So, we can write the nodal equilibrium equation just by inspection of the ckt, no need to write the eq by applying KCL.

x. Apply the nodal analysis to the below given circuit and write the equilibrium equation in matrix form.



- This network has total 5 nodes , i.e. n+1=5, & n=4.
- The four Real nodes are A,B,C & D.
- The 5th node, E is datum node.
- We have to write FOUR Equilibrium Equations for the nodes A, B, C and D using KVL.
- Here we shall use all rules and assumption, we have made previously.
- The first node we take is "A" , connecting R_1, R_2, L_1, $i_1(t)$ & $i_2(t)$. So by KCL,

$$\frac{V_A}{R_1} + \frac{V_A - V_B}{R_2} + \frac{1}{L_1}\int_{-\infty}^{T}(V_A - V_B)\,dt - i_1(t) + i_2(t) = 0$$

Changing lower limit of integration from -∞ to 0 , using p operator and rearranging the coefficients of V_A, V_B, V_C & V_D,

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{L_1p}\right)V_A+\left(-\frac{1}{R_2}-\frac{1}{L_1p}\right)V_B+0\times V_C+0\times V_D=i_1(t)-i_2(t) \quad \text{---------(1)}$$

- Now the equilibrium eq at node B which connects, R_2, L_1, L_2, C_1, & C_2 & no any source.

$$\frac{V_B-V_A}{R_2}+\frac{1}{L_1}\int_{-\infty}^{T}(V_B-V_A)dt+\frac{1}{L_2}\int_{-\infty}^{T}(V_B-V_C)dt-C_1\frac{dV_B}{dt}+C_1\frac{d(V_B-V_C)}{dt}=0$$

$$\frac{V_B-V_A}{R_2}+\frac{1}{L_1}\int_{0}^{T}(V_B-V_A)dt+\frac{1}{L_2}\int_{0}^{T}(V_B-V_C)dt-C_1\frac{dV_B}{dt}+C_1\frac{d(V_B-V_C)}{dt}=0$$

- Here also Changing lower limit of integration from -∞ to 0 , using p operator and rearranging the coefficients of V_A, V_B, V_C & V_D,

$$-\left(\frac{1}{R_2}-\frac{1}{L_1p}\right)V_A+\left(\frac{1}{R_2}+\frac{1}{L_1p}+\frac{1}{L_2p}+C_1p+C_2p\right)V_B+\left(-C_2p-\frac{1}{L_2p}\right)V_C+0\times V_D=0 \quad \text{---------(2)}$$

Same thing for node C gives,

$$\frac{V_C}{R_3}+\frac{1}{L_2}\int_{-\infty}^{T}(V_C-V_B)dt+C_2\frac{d(V_C-V_B)}{dt}+C_3\frac{d(V_C-V_D)}{dt}=0$$

- The simplified equation at node C,

$$0\times V_A+\left(-C_2p-\frac{1}{L_2p}\right)V_B+\left(\frac{1}{R_3}+\frac{1}{L_2p}+C_2p+C_3p\right)V_C+(-C_3p)\times V_D=0 \quad \text{---------(3)}$$

For node D, by KCL,

$$\frac{1}{L_4}\int_{-\infty}^{T} V_D\,dt + C_3\frac{d(V_D - V_C)}{dt} - i_2(t) - i_2(t) = 0$$

- In simplified form,

$$0 \times V_A + 0 \times V_B + (-C_3p)V_C + \left(\frac{1}{L_3p} + C_3p\right)V_D = i_2(t) + i_3(t) \quad \text{------------(4)}$$

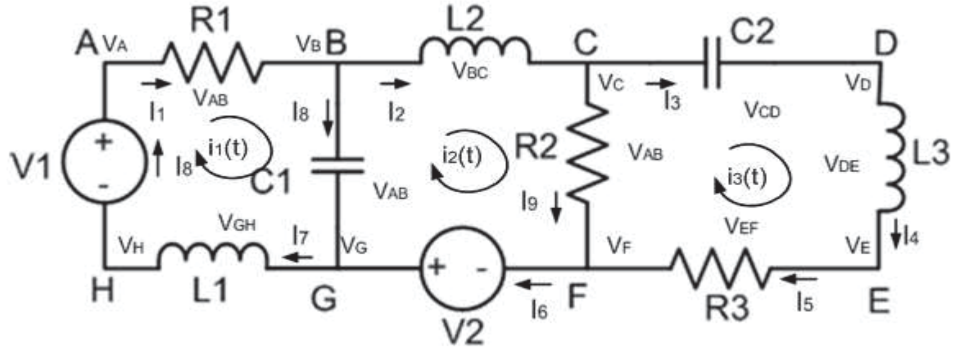
- So, in the matrix form,

$$\begin{bmatrix} \left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{L_1p}\right] & \left[-\frac{1}{R_2}-\frac{1}{L_1p}\right] & 0 & 0 \\ \left[-\frac{1}{R_2}-\frac{1}{L_1p}\right] & \left[\frac{1}{R_2}+\frac{1}{L_1p}+\frac{1}{L_2p}+C_1p+C_2p\right] & \left[-C_2p-\frac{1}{L_2p}\right] & 0 \\ 0 & \left[-C_2p-\frac{1}{L_2p}\right] & \left[\frac{1}{R_3}+\frac{1}{L_2p}+C_2p+C_3p\right] & \left[-C_3p\right] \\ 0 & 0 & \left[-C_3p\right] & \left[\frac{1}{L_3p}+C_3p\right] \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} i_1(t)-i_2(t) \\ 0 \\ 0 \\ i_2(t)+i_3(t) \end{bmatrix}$$

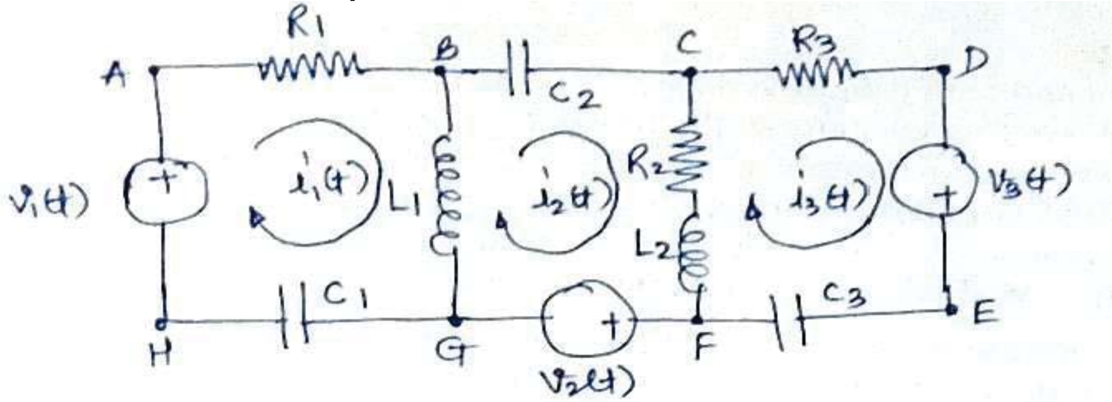
MESH ANALYSIS:

- Next, we shall consider the case of finding the set of independent currents of a given circuit.
- These are the currents around every independent mesh, called **mesh currents**.
- The meshes forming the independent closed contours inside the ckt are called as the **inner meshes** (analogous to real nodes).
- The loop covering the outer boundaries, called **outer mesh/ loop** (anologous to the reference node).
- If the inner meshes are "n" in number , the **total meshes are n+1**.
- The analysis used to find these mesh currents is called as "**MESH ANALYSIS**".
- First we count the number of independent meshes, "n",
- We shall apply the KVL in each mesh and collect all the voltage drops & raises, meeting in each mesh (mesh currents are different than the branch currents).
- Here also, we make some assumptions.

1. We shall go in clockwise direction and the "round arrow" will show the direction of this mesh current.
2. The current of the mesh, where we are, is greater than the other currents
3. The voltage drops shall be taken as +ve and the voltage raises to be taken as -ve.



x. We shall solve an example to illustrate the method.



Following points we shall note here.

1. The no. of inner meshes are , m = 3. They are,
   - Mesh- 1: ABGHA (with mesh current $i_1(t)$),
   - Mesh- 2: BCFGB (with mesh current $i_2(t)$),
   - Mesh- 3: CDEFC (with mesh current $i_3(t)$).
2. The outer loop is : ABCDEFGHA (carries all mesh currents.).

We shall take every mesh, one by one.

Mesh 1: By applying the KVL around this mesh, we get the voltages as,

1. Drop across R_1 (taken +ve) = $R_1 i_1(t)$
2. Drop across L_1 (taken +ve) = $L_1 \frac{d}{dt}(i_1(t) - i_2(t))$
3. Drop across C_1 (taken +ve) = $\frac{1}{C_1}\int_{-\infty}^{t} i_1(t)\,dt$
4. Volage of voltage sourse $v_1(t)$ (taken -ve) = $-v_1(t)$

Thus the final equation using the KVL in mesh-1 is,

$$R_1 i_1(t) + L_1 \frac{d}{dt}\left(i_1(t) - i_2(t)\right) + \frac{1}{C_1}\int_{-\infty}^{t} i_1(t)\,dt - v_1(t) = 0$$

- Here too, there is the integration term written with limits -∞ to t.. It will be broken into two parts.

$$\frac{1}{C_1}\int_{-\infty}^{t} i_1(t)\,dt \;=\; \frac{1}{C_1}\int_{-\infty}^{t} i_1(t) + dt\,\frac{1}{C_1}\int_{0}^{t} i_1(t)\,dt$$

- The first term is the prior history of the charge stored by capacitor (initial voltage on capacitor) before the analysis starts and the second is the regular term as usual.
- We assume the initial chare to be zero. We use p-operator here.

$$R_1 i_1(t) + L_1 p\left(i_1(t) - i_2(t)\right) + \frac{1}{C_1 p} i_1(t) = v_1(t)$$

- And finally, we have the most simplified form of the equilibrium eq of mesh-1

$$(R_1 + \frac{1}{C_1 p} + L_1 p) i_1(t) + (-L_1 p)\left(i_2(t)\right) + 0 \times i_3(t) = v_1(t) \qquad \text{---------(1)}$$

- Now applying KVL in mesh-2 we get

$$L_1 \frac{d}{dt}\left(i_2(t) - i_1(t)\right) + \frac{1}{C_2}\int_{0}^{t} i_2(t)\,dt + R_1\left(i_2(t) - i_3(t)\right) + L_1 \frac{d}{dt}\left(i_2(t) - i_3(t)\right) + v_2(t) = 0$$

- With use of p-operator,

$$L_1 p\left(i_2(t) - i_1(t)\right) + \frac{1}{C_2 p} i_2(t) dt + R_2\left(i_2(t) - i_3(t)\right) + L_2 p\left(i_2(t) - i_3(t)\right) = -v_2(t)$$

For Equilibrium Eq of mesh -2, collecting the coefficients of the unknown variables, here, i1(t), i2(t) and i3(t), we get,

$$(-L_1p)i_1(t) + (L_1p + \frac{1}{C_2p} + L_2p + R_2)i_2(t) + (-R_2 - L_2p)i_3(t) = -v_2(t) \quad \text{--------(2)}$$

- Similarly, we write the equilibrium equation for mesh-3.

$$L_2\frac{d}{dt}(i_3(t) - i_2(t)) + R_2(i_3(t) - i_2(t)) + R_3i_3(t) + \frac{1}{C_3}\int_0^t i_3(t)\,dt = v_3(t)$$

Simplifying gives the equilibrium eq in the terms of 3 unknown variables $i_1(t)$, $i_2(t)$ and $i_3(t)$, we get,

$$0 \times i_1(t) + (-R_2 - L_2p)i_2(t) + (L_2p + R_2 + R_3 + \frac{1}{C_3p})i_3(t) = v_3(t) \quad \text{------------(3)}$$

$$\begin{bmatrix} (R_1 + \frac{1}{C_1p} + L_1p) & (-L_1p) & 0 \\ (-L_1p) & (L_1p + \frac{1}{C_2p} + L_2p + R_2) & (-R_2 - L_2p) \\ 0 & (-R_2 - L_2p) & (L_2p + R_2 + R_3 + \frac{1}{C_3p}) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ -v_2(t) \\ v_3(t) \end{bmatrix}$$

In this 3 x 3 matrix all entries are impedances and it's a square matrix, which is called as the **Mesh Impedance Matrix.**

So, it follows the rule that, $[Z] \times [I] = [V]$

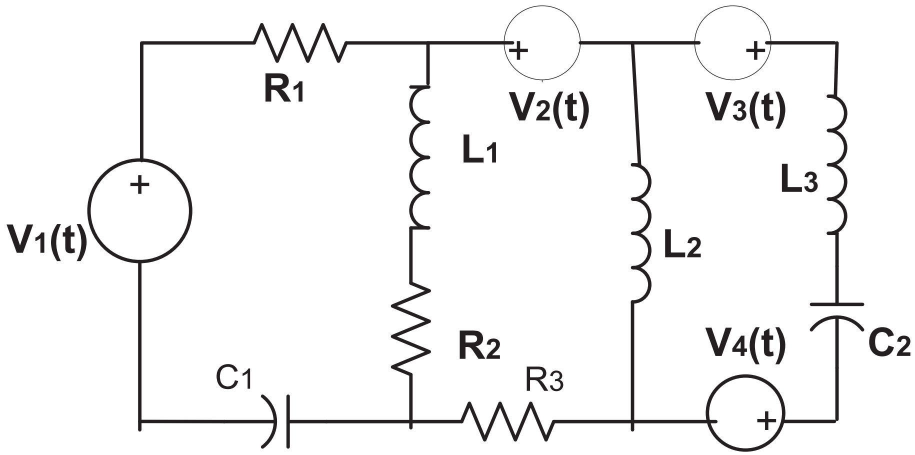
$[Z]$ is nxn matrix, $[I]$ is nx1 i.e. column matrix and $[V]$ also is nx1 i.e. column matrix.

$[Z]$ is **Mesh Impedance Matrix** could be shown as,

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

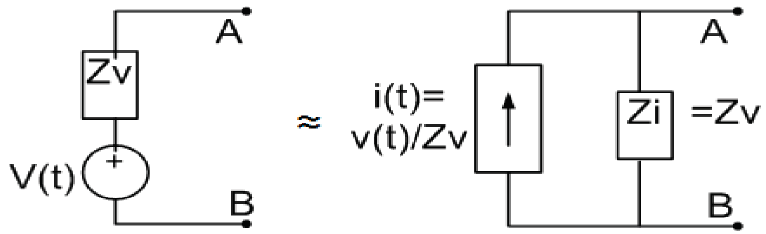
- The entries with same suffices i.e. z_{11}, z_{22} and z_{33} are called as the **diagonal entries**.
- And the entries with different suffices i.e. z_{12}, z_{13}, ..z_{32} are called as **off- diagonal entries**.
- The diagonal entries, 'z_{kk}' are the positive sum of the impedances of all the branches meeting in mesh-1, they are called as the **self - impedances** of mesh-1.
- The off-diagonal entries 'z_{jk}' are, the negative sums of impedances of the branches connected between the mesh-1 in hand and all other neighboring meshes. These impedances are called as the **mutual impedances.**
- The off-diagonal entries 'z_{jk}' follow the rule that $z_{jk} = z_{kj}$ i.e. $z_{12} = z_{21}$, $z_{23} = z_{32}$ etc.. This is entirely due to the fact that, we are assuming, the circuit is connected with only linear & bilateral elements.
- So, we can write the mesh equilibrium equation just by inspection of the ckt, no need to write the eq by applying KCL.

Ex: Write the equilibrium equation in matrix form.



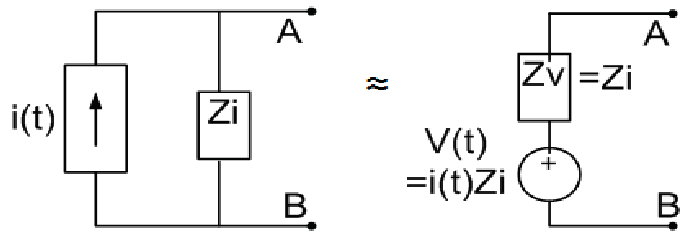
›urce Transformation

- ‚Ve have analyzed the n/ws connecting current sources by nodal analysis and the n/ws connecting voltage sources by mesh analysis.
- But, many times we find voltage sources in a node analysis n/w and the current sources in the mesh analysis n/w which are quite uncomfortable for solving.
- Hence we use the source transformation, where voltage source can be transformed to the current source as,



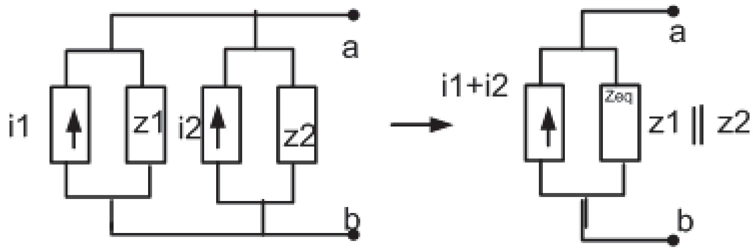
Here, Zi = Zv
& i(t)= v(t) / Zv

And current source can be converted to a voltage source as,

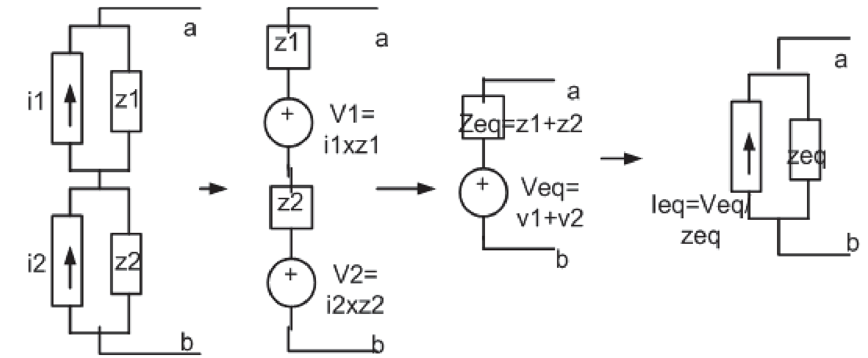


Here, Zv = Zi
& V(t)= i(t) x Zi

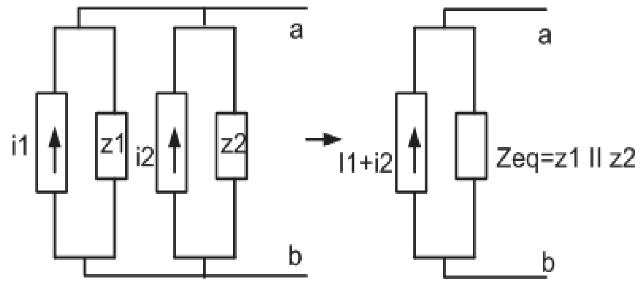
Two Current Sources in parallel



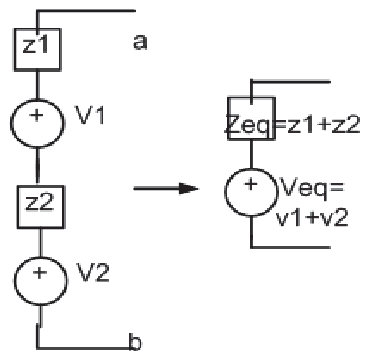
Two Current Sources in Series:



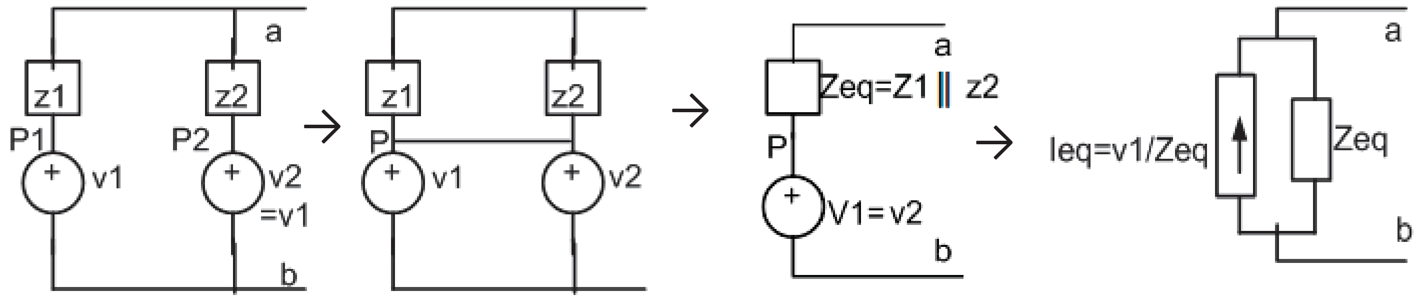
- Two Current Sources in Parallel:



- Two Voltage Sources in Series:

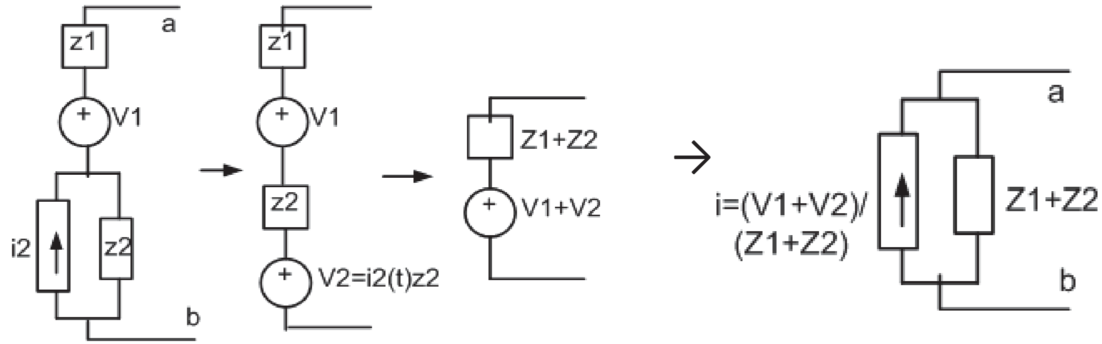


Two Voltage Sources in Parallel:



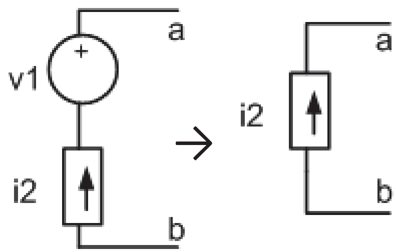
- The two terminals P1 & P2 are at same potential, equipotential, hence drawn as a single terminal.

❖ Current & Voltage sources (practical) in Series:



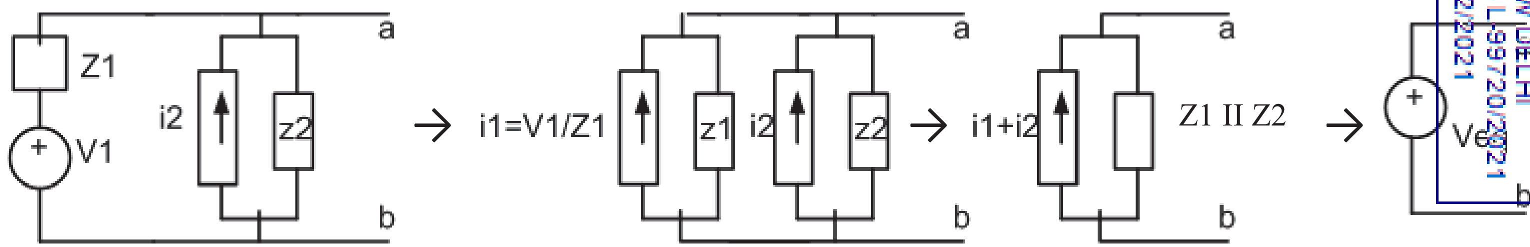
$$i = \frac{v_1 + i_2 z_2}{z_1 + z_2}$$

❖ Current & Voltage sources (ideal) in Series: Its equivalent to the single current source only.



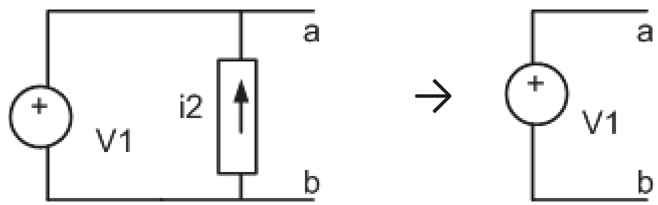
$$i = \lim_{\substack{z_1 \to 0 \\ z_2 \to \infty}} \frac{v_1 + i_2 z_2}{z_1 + z_2} = i_2$$

Current & Voltage sources (practical) in Parallel:



$$\text{Veq} = \frac{\frac{v_1}{z_1} + i_2}{z_1 \| z_2} = \frac{v_1 z_2 + i_2 z_1 z_2}{z_1 + z_2}$$

❖ Current & Voltage sources (Ideal) in Parallel:

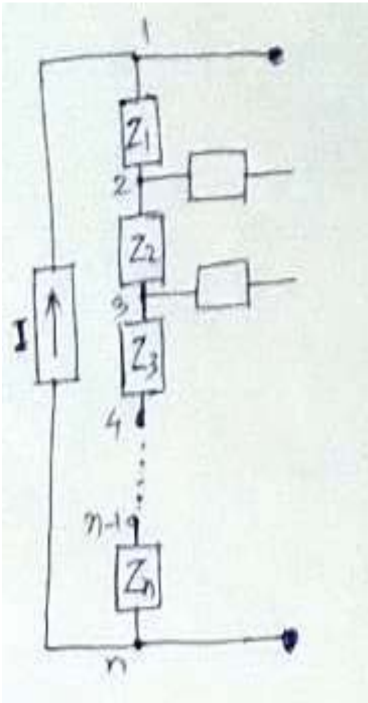


$$\lim_{\substack{z_1 \to 0 \\ z_2 \to \infty}} \frac{\frac{v_1}{z_1} + i_2}{z_1 \| z_2} = \lim_{\substack{z_1 \to 0 \\ z_2 \to \infty}} \frac{v_1 z_2 + i_2 z_1 z_2}{z_1 + z_2} = v1$$

Some Other Combinations:

I-Shift :

- If a current source is connected at nodes 1 and n as shown, connecting many impedances Z1, Z2, Z3,,,Zn between the inner nodes, then this current source could be shown connected As in the figure below.
- Beacause there is no current changed with respect to the magnitude and direction, whatever current enters in every node same leaves at the same time.



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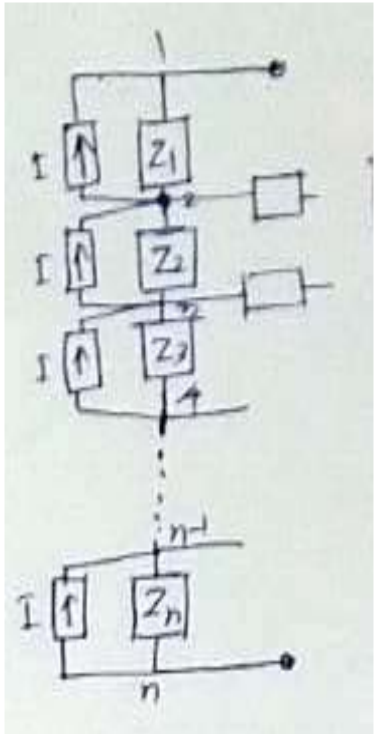
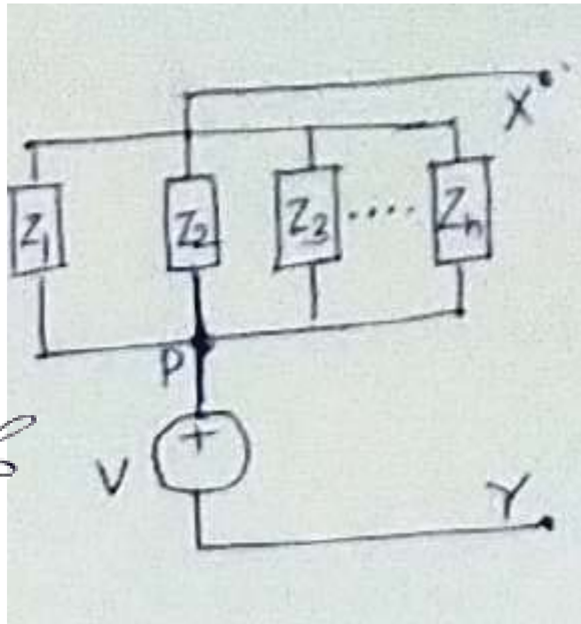


Shift :

f a voltage source is connected in series to the parallel combination of impedances Z1, Z2, Z3,…Zn, at one of their end terminal like P here and Q the lower terminal, then this is equivalent to the connection of one distinct voltage source to each of the impedances in series.

- This is because the terminals P1, P2, P3,…Pn are equipotential, same as P. So, shifting the voltage source toward the impedances past these terminals, is not wrong.



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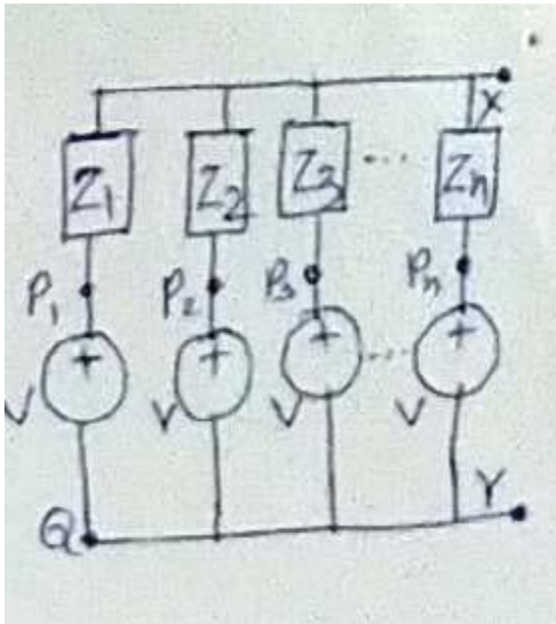
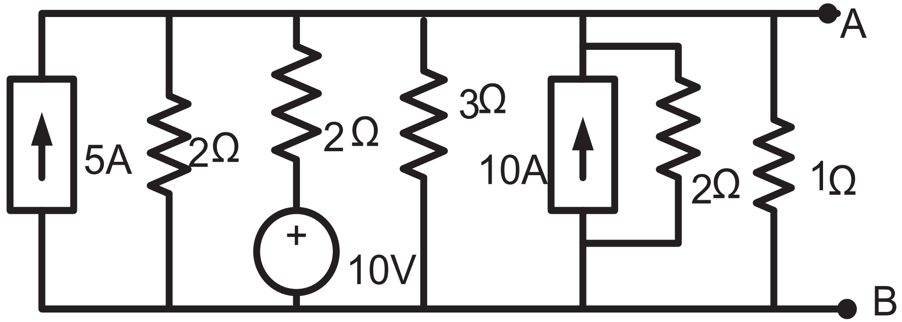
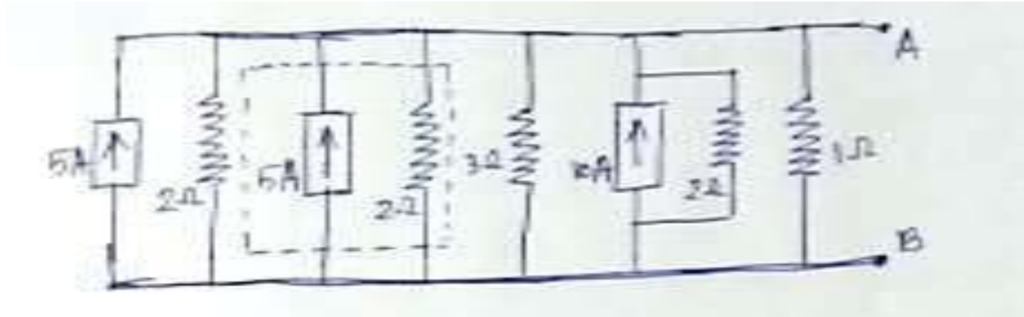


Using source transformation, convert the following network into a single voltage source and a single resistance between A & B.





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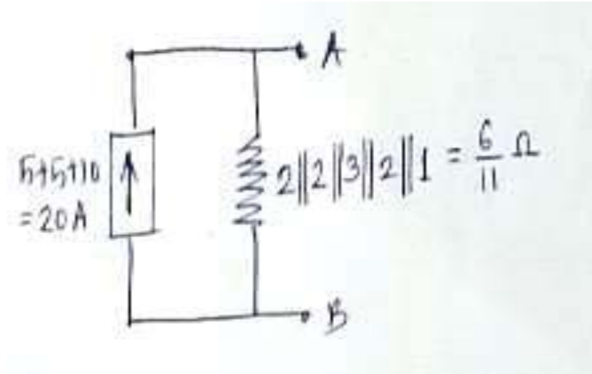

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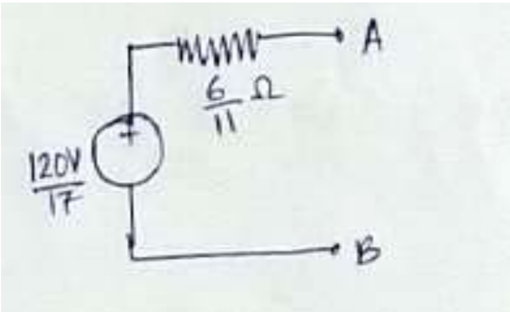
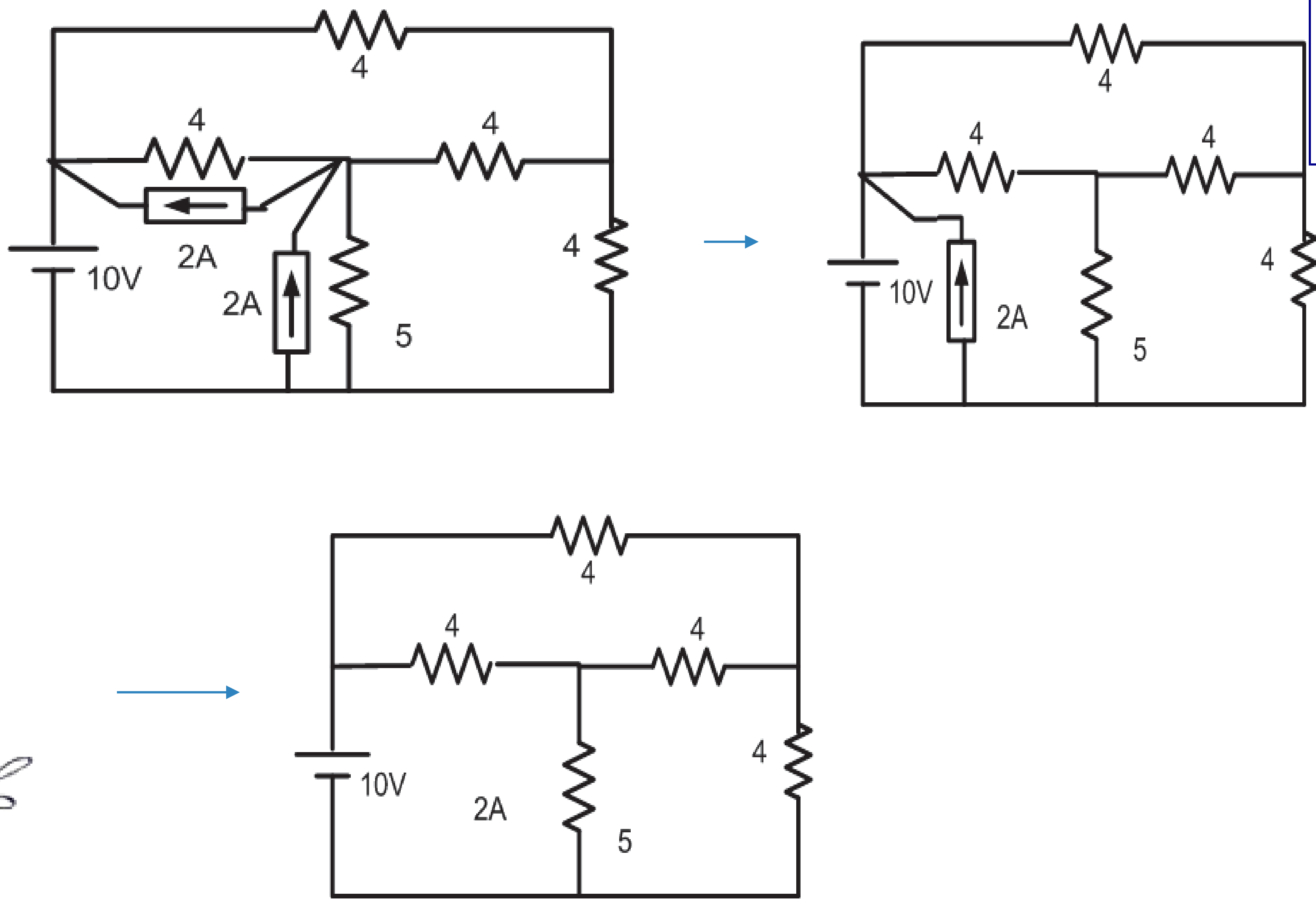


Ex. Find the current through 5 ohm resistor.



$$\begin{bmatrix} 4+5 & -5 & -4 \\ -5 & 5+4+4 & -4 \\ -4 & -4 & 4+4+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

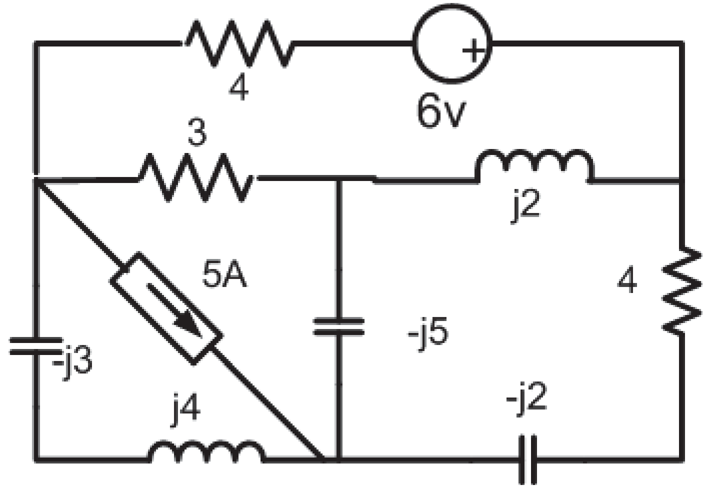
Using Cramer's rule,

$$i_1 = \frac{\begin{bmatrix} 10 & -5 & -4 \\ 0 & 13 & -4 \\ 0 & -4 & 12 \end{bmatrix}}{\begin{bmatrix} 9 & -5 & -4 \\ -5 & 13 & -4 \\ -4 & -4 & 12 \end{bmatrix}} = 2.36\text{ A}$$

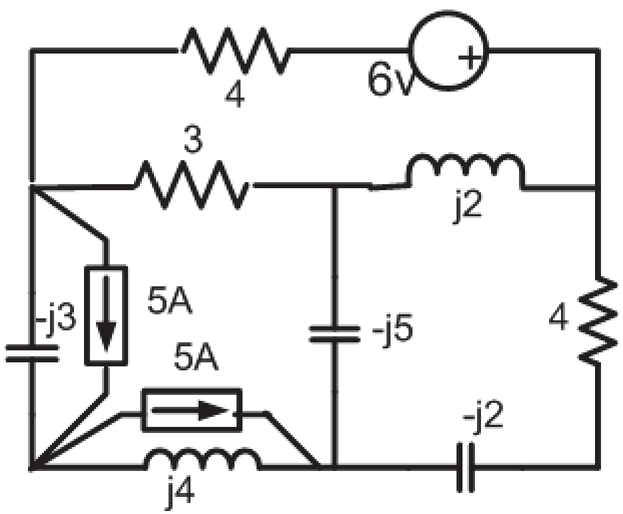
$$i_2 = \frac{\begin{bmatrix} 9 & 10 & -4 \\ -5 & 0 & -4 \\ -4 & 0 & 12 \end{bmatrix}}{\begin{bmatrix} 9 & -5 & -4 \\ -5 & 13 & -4 \\ -4 & -4 & 12 \end{bmatrix}} = 1.28\text{A}$$

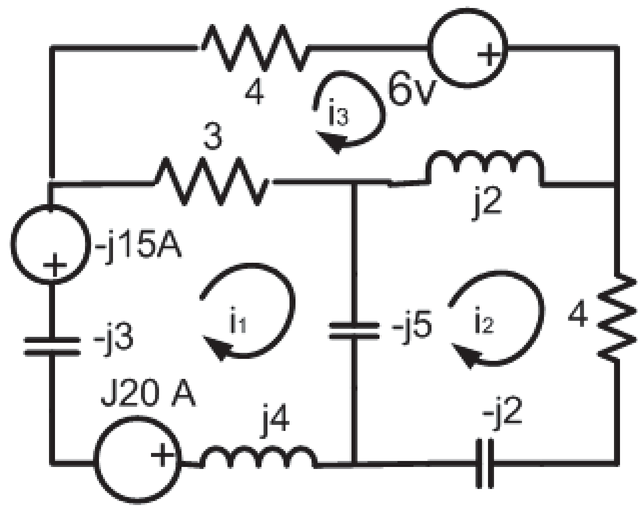
Hence, Current through 5 ohm resistance = 2.36-1.28= 1.08 A

Ex. Write the equilibrium equation for the given network in matrix form.

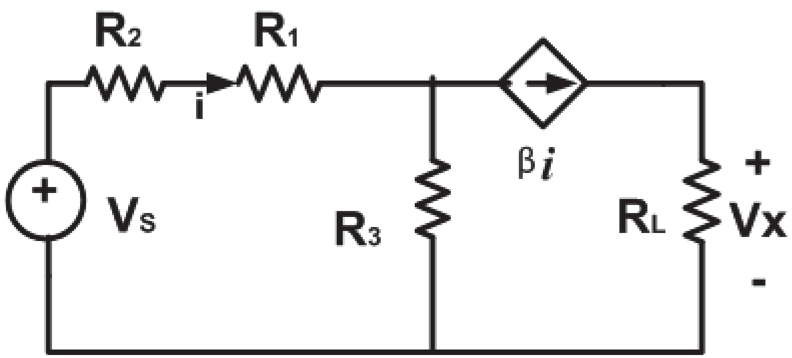


Sol: By I-shift and source transformation





x. For the circuit of following Fig, find Vx in terms of Vs.



Sol:

By KVL in mesh 1 & 2,

$$-V_s + i(R_1 + R_2) + R_3(i - \beta i) = 0 \quad \text{---(1)}$$

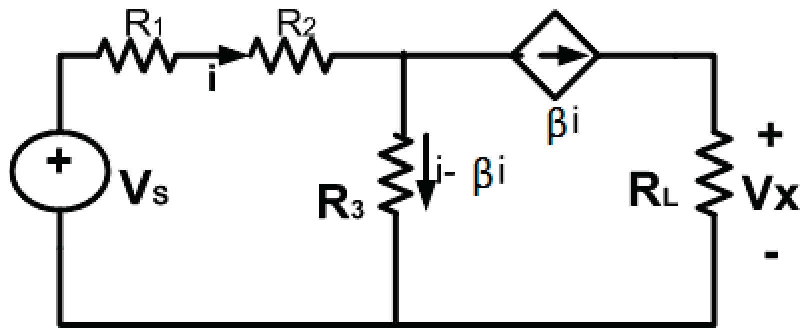
$$-R_3(i - \beta i) + V_x = 0 \rightarrow R_3(i - \beta i) = V_x \quad \text{---(2)}$$

Put (2) in (1), $i(R_1 + R_2) + V_x = V_S$

$$V_x = V_S - i(R_1 + R_2)$$

But, $V_x = \beta i R_L$ So, $i = \frac{V_x}{\beta R_L}$

Hence, $V_x = V_S - \frac{V_x}{\beta R_L}(R_1 + R_2)$



$$V_x = V_S - V_x \frac{(R_1 + R_2)}{\beta R_L} \rightarrow V_x + V_x \frac{(R_1 + R_2)}{\beta R_L} = V_S$$

$$V_S = V_x \left(1 + \frac{(R_1 + R_2)}{\beta R_L}\right)$$

Thus, Vx in terms of Vs is,

$$V_x = V_s \left(\frac{\beta R_L}{\beta R_L + R_1 + R_2}\right)$$

agnetically Coupled Coils With Dot Markings

The changing magnetic flux due to the changing current in one coil, if linked with another coil connected close to it, can produce the induced EMF in that another coil, according to the **“Faraday’s Law of Electromagnetic Induction”.**

- Such coils are called as, magnetically coupled coils or just **“coupled coils”.**

- For knowing the magnitude and directions of all such emfs / voltages correctly, across the neighboring coils, it is necessary to know the direction of currents (entering/leaving) with the coils. The relation of these directions of currents and polarity of such emfs are fixed by these dots. Hence **Dot Markings** are important for the correct calculations of voltages of a circuit.

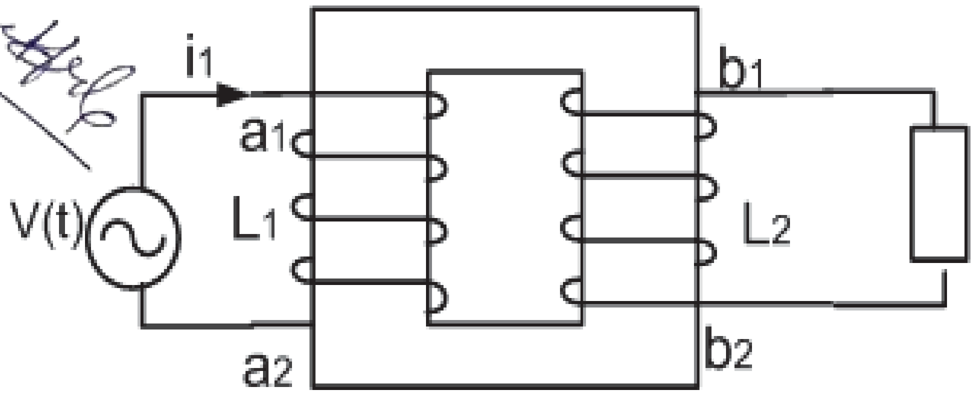
- The positions of the Dot Marks depends on the directions of currents (hence magnetic field) and the winding sense (how it is wound on the core).

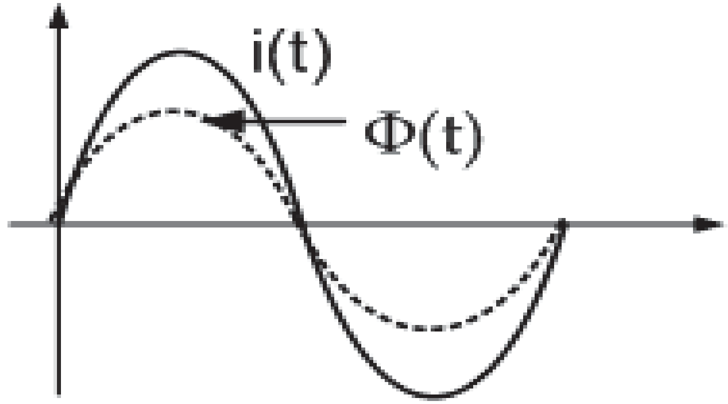
Mutually Coupled Coils and Lenz's Law

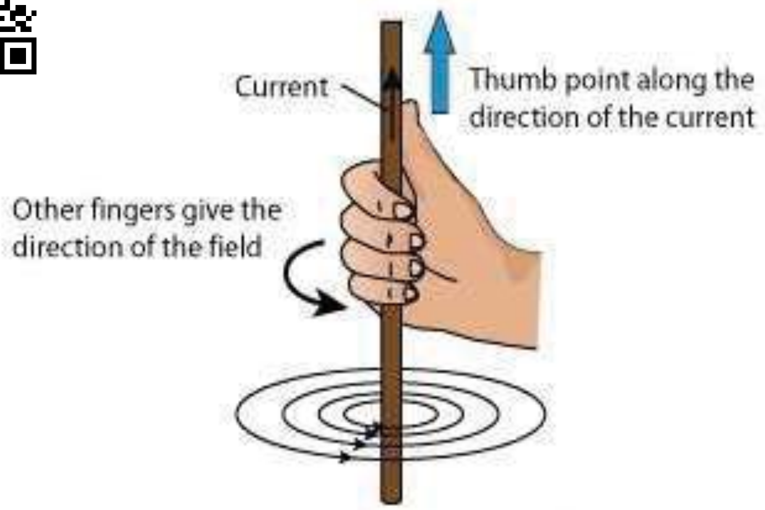
Consider that two inductive coils L_1 & L_2 are placed on the common magnetic core (like transformer core)

- Coil L_1 fixed on left side between terminals a_1 and a_2, connected to a source of sinusoidally varying voltage v(t), so it's current i1(t) also sinusoidally varying which creates the magnetic flux, Φ_1 (t) (also sinusoidally varying).

- The direction of the flux lines around the current carrying conductor can be decided using, **Maxwell's Right Hand Grip Rule Or Cork Screw Rule**.







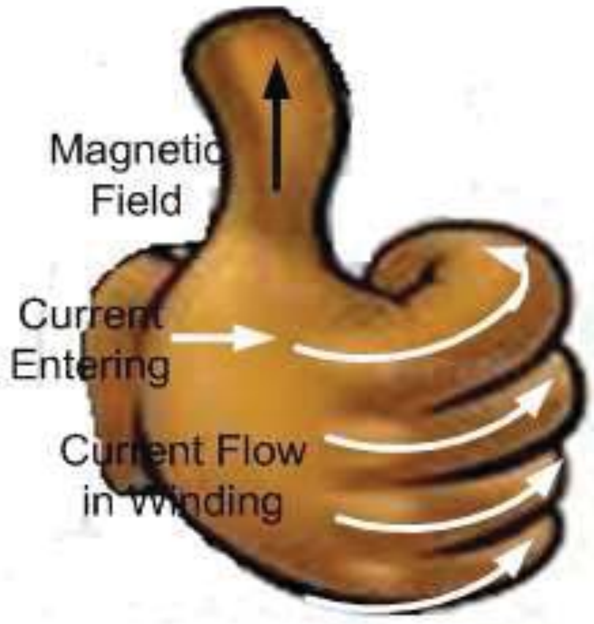
Right Hand Grip Rule

Maxwell's right hand grip rule

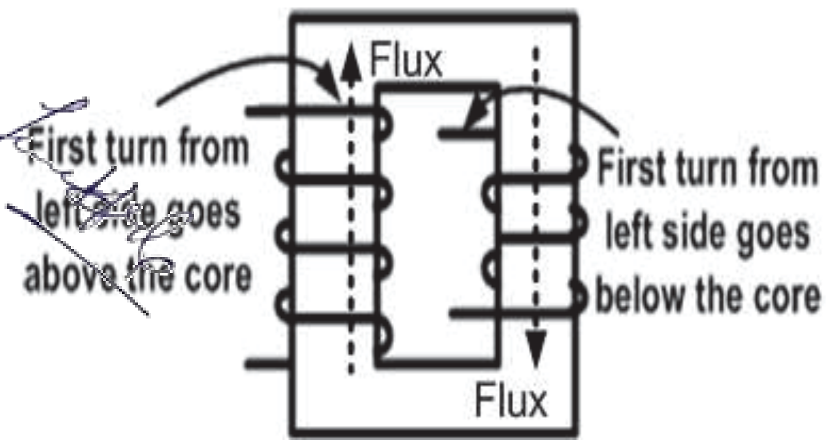
Assume that the current carrying conductor is held in the right hand so that the fingers wrap around the conductor and the thumb is stretched (as shown in the figure at left). If the thumb is along the direction of current, wrapped fingers will show the direction of circular magnetic field lines.

- Thus, if the above rule applied in reverse, gives the direction of the combined magnetic flux (called as, **Right Hand Rule** also), which state that, if fingers are wrapped around the coil with fingers pointed in the direction of the current, the thumb of the right hand indicates the direction of the flux inside the coil (passing through the core).

- With respect to our figure of above transformer, this means that, if we are wrapping a conductor in anti-clockwise direction from left side of core, so that it's first turn is running above the core, the magnetic flux will flow **UPWARD**.



Right Hand Rule



PRACTICAL UNDERSTANDING:

- ❖ If current enters the first turn (upper turn of the coil which goes **above the core** from left side, then the combined flux will **go UPWARD** through the core limb.
- ❖ And if current enters the first turn (upper turn) of the coil which goes **below the core** from left side, then the combined flux in the core limb will **go DOWNWARD**.
- ❖ The flux due to positively increasing current i_1 will thus generate flux Φ_1 upward in the left limb & clockwise in whole core.
- ❖ This sinusoidally changing flux links the second coil L_2 as well as it's own coil L_1.
- ❖ Thus it induces the emfs e1 across it's own coil L1 and e2 across the neighboring coil L2.

- When coil L1 carries positively increasing current and enters through a1, the emf i induces across it's own coil L1, is called as the **Self Induced Emf**. Given as ,

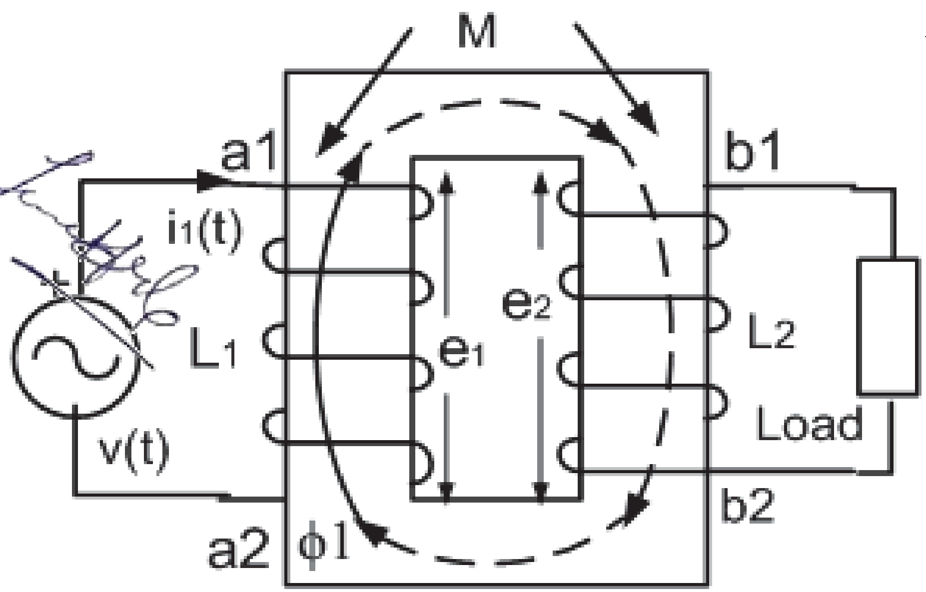
$$e_{L1} = -L_1 \frac{di1}{dt}$$

- It is shown –ve as it is in opposite direction to the supply voltage.
- This is because it follows the Lenz's Law.
- Lenz's Law states that , 'the induced emf across the coil will be created in such a direction that it will always try to vanish the very cause of its generation' .
- This means that, the emf e2 will be induced in coil L2 and will create the current i2 which ultimately generate the magnetic flux ϕ2, in such a direction that the cause of its generation i.e. magnetic flux ϕ1 due to i1 will be vanished.
- So the coil L2 will produce the flux ϕ2 , equal in magnitude & opposite in direction to ϕ1.
- This will induce emf across coil L2, called as the **Mutually Induced Emf,** given as,

$$e_M = \pm M \frac{di1}{dt}$$ (As it is due to current i1.)

So, this means that when a timely varying current passes through a coil, then it creat the self induced emf in its own coil and mutually induced emf in the coupled coil if any there.

- Same thing will happen in the same way when current i2 will flow through the coil L2, it will create self induced emf e_{L2} , in its own coil and the mutually induced emf e_M in coil L1.
- Therefore, each coil will hold two emf in total. They are,



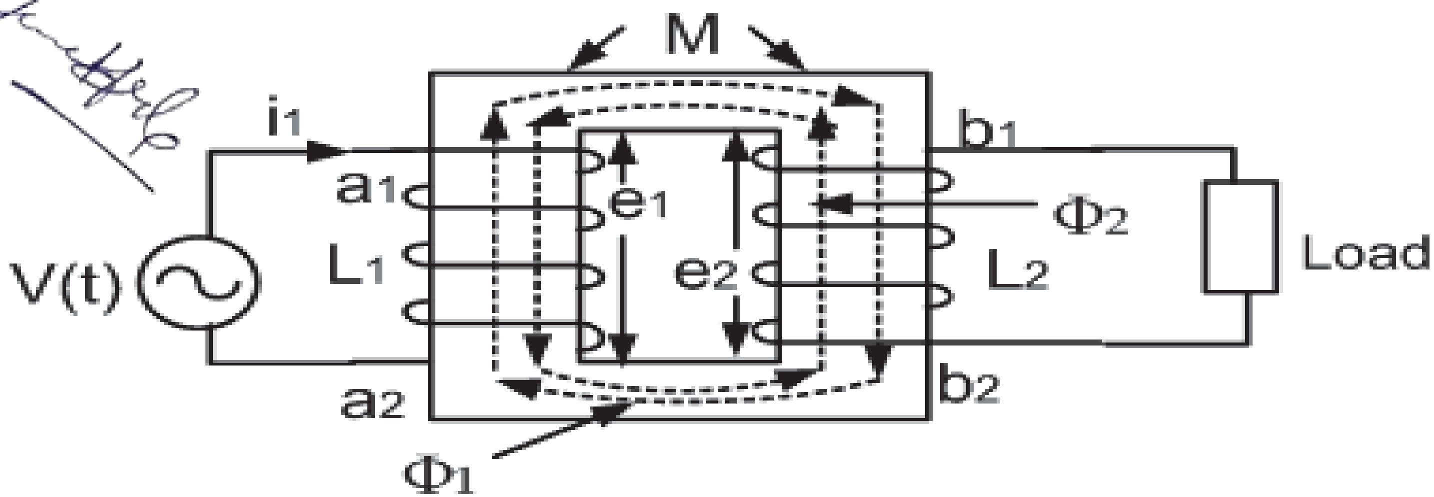
Across coil L1, $e_1 = -L_1 \frac{di1}{dt} \pm M \frac{di2}{dt}$

Across coil L1,

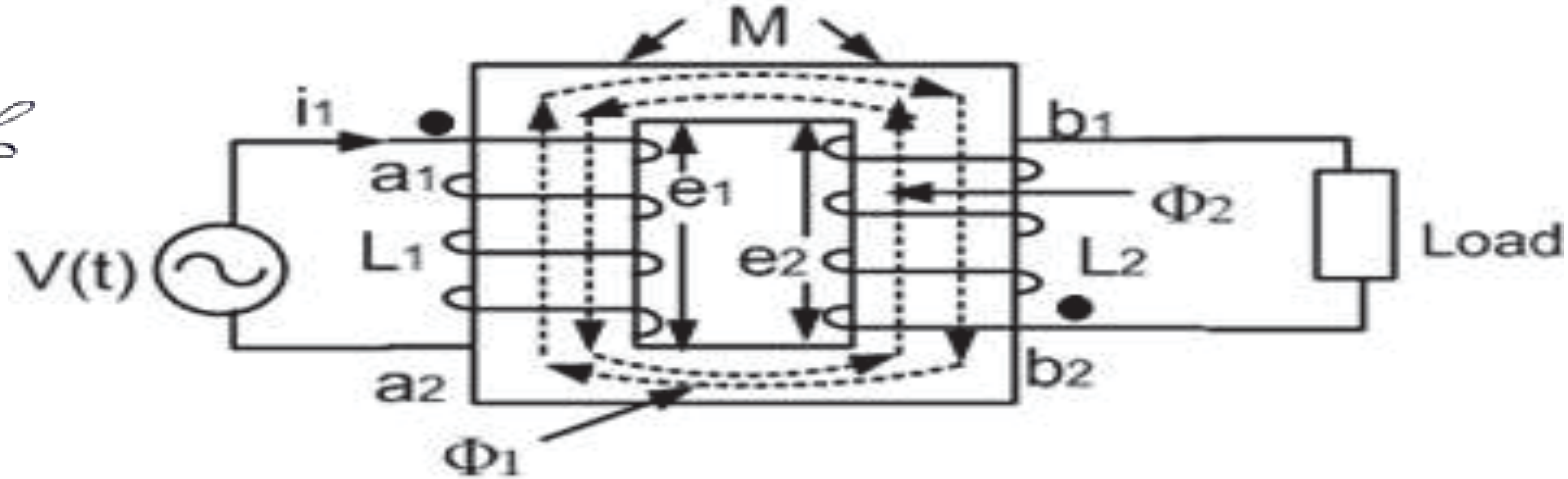
$$e_2 = -L_2 \frac{di2}{dt} \pm M \frac{di1}{dt}$$

- $\pm M$ of mutual induction suggests that the addition / subtraction of this emf depends on the dot marking and direction of magnetic field in the core.

- The coefficient of mutual inductance due to current i1 of L1 on coil L2 is M12 and the coefficient of mutual inductance due to current i2 of L2 on coil L1 is M21, but they are taken here equal, i.e. M12 = M21 = M = $\sqrt{L_1 L_2}$,
- Generally it is, M = k $\sqrt{L_1 L_2}$, where k is the coefficient of magnetic coupling, k <1.
- Thus summarily, when voltage v(t) is positive at the upper end, positively increasing current i1 enters the coil L1 through terminal a1 and produces the upward magnetic flux ϕ1 in clockwise direction. It gets linked with its own coil L1 and the second coil L2. Due to Lenz's law, both coil will respond by producing the opposite directional i.e. anticlockwise flux ϕ2 (by Right Hand Rule). This flux ϕ2, would have been produced by e2, which forces the current i2 to enter the coil L2 through b1 (by Right Hand Rule).



- We here put a dot '●' at **a1 as it is positive here**.
- We assume that, we shall place a dot same as this, on another (coupled) coil which goes positive at that time.
- And that terminal on L2 is the lower terminal b2, from where the current i2 comes out. Terminal **b2 is thus positive and we place the dot at it.**
- We see here that, when the current i2 enters though the terminal b1, $\phi 2$ (anticlockwise) opposes $\phi 1$.
- Therefore, these fluxes are subtractive and the induced emf e_M is also subtractive.
- Alternatively, if i2 enters through the terminal b2, flux $\phi 2$ will be clockwise and both will be additive to each other. Thus, e_M also additive.



Importantly here, the coil L2 is working as a generator of emf, it will be positive at the end from where the current leaves i.e. comes out.

Thus if current i1 enters through a1, dotted terminal and current i2 enters through b1, undotted terminal the emfs generated in coil L1 & L2 will be subtractive,

$$e_1 = -L_1\frac{di1}{dt} + M\frac{di2}{dt} \quad \text{And} \quad e_2 = -L_2\frac{di2}{dt} + M\frac{di1}{dt}$$

- Hence the supply voltages v1 & v2 respectively, to oppose this is,

$$v_1 = +L_1\frac{di1}{dt} - M\frac{di2}{dt} \quad \text{And} \quad v_2 = +L_2\frac{di2}{dt} - M\frac{di1}{dt}$$

- Similarly, if current i1 enters through a1, dotted terminal and current i2 enters through b2, also dotted terminal (both enters through the dotted terminals) then the emfs generated in coil L1 & L2 will be additive,

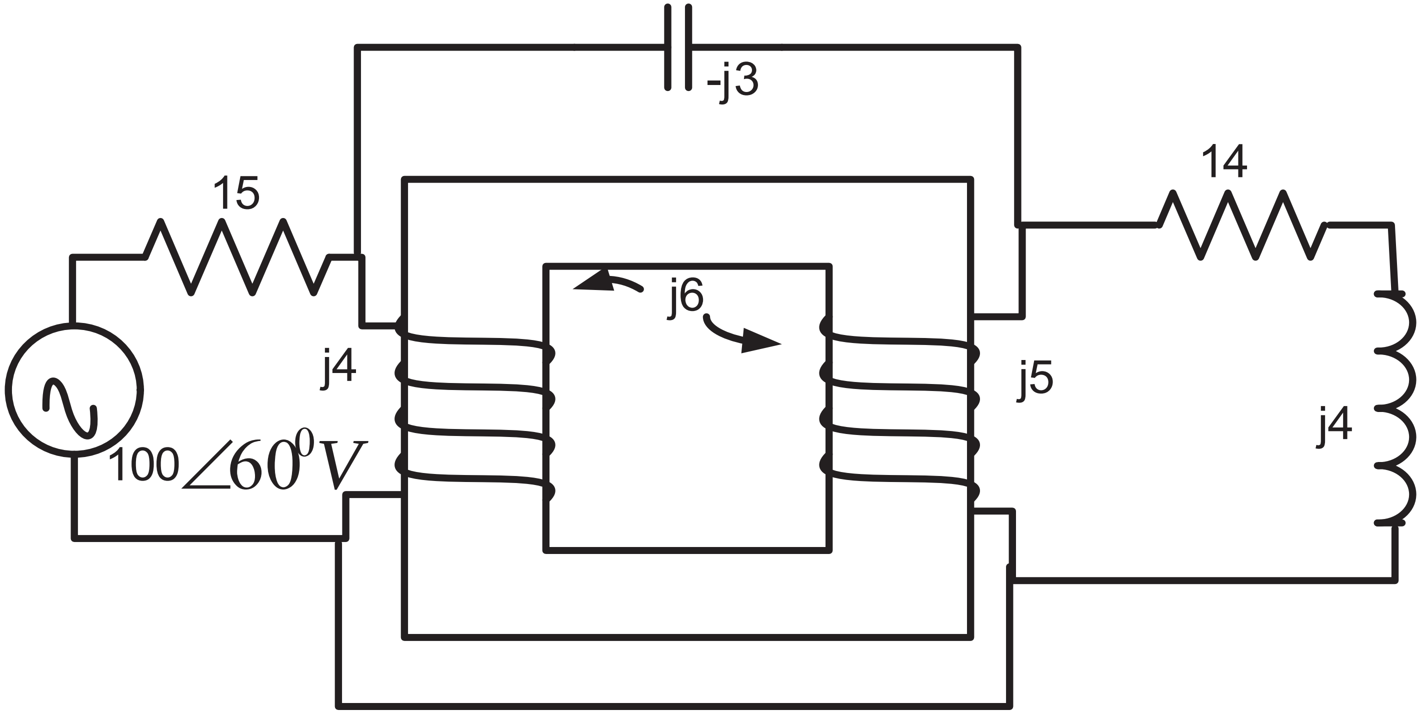
$$e_1 = -L_1\frac{di1}{dt} - M\frac{di2}{dt} \quad \text{And} \quad e_2 = -L_2\frac{di2}{dt} - M\frac{di1}{dt}$$

- And the opposing voltages are,

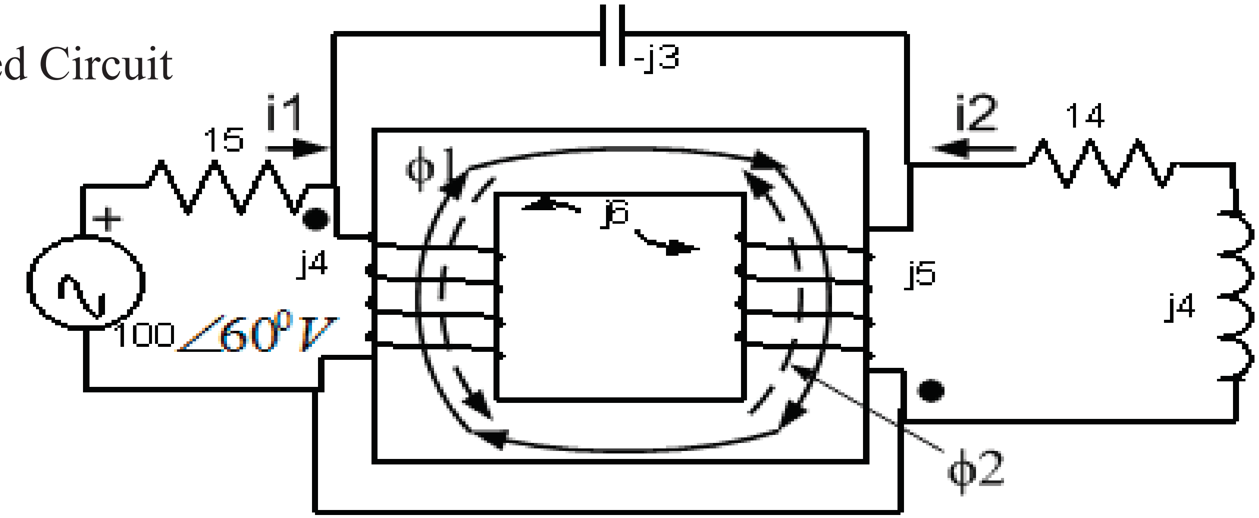
$$v_1 = +L_1\frac{di1}{dt} + M\frac{di2}{dt} \quad \text{And} \quad v_2 = +L_2\frac{di2}{dt} + M\frac{di1}{dt}$$

- Thus, for one coupled coil pair a dot like '●' can be used and the equation for v1 & v2 Could be written as above. If there are more than one pairs, dots like, ♠, ♣, ♥, ♦ could be used.

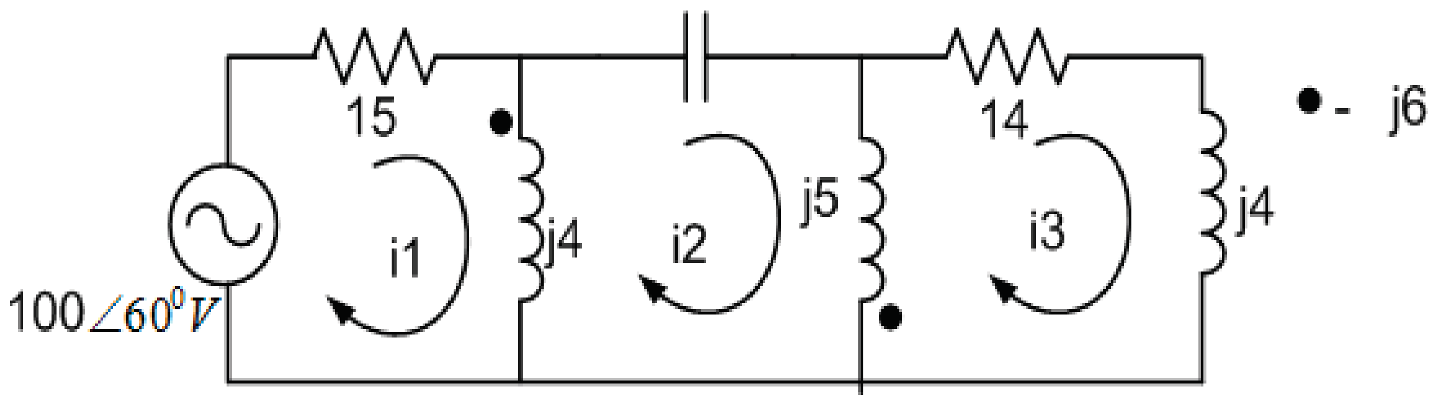
Ex. Mark the dots on the circuit for mutual coupling, redraw the dotted equivalent circuit and find voltage across inductance j4.5Ω.



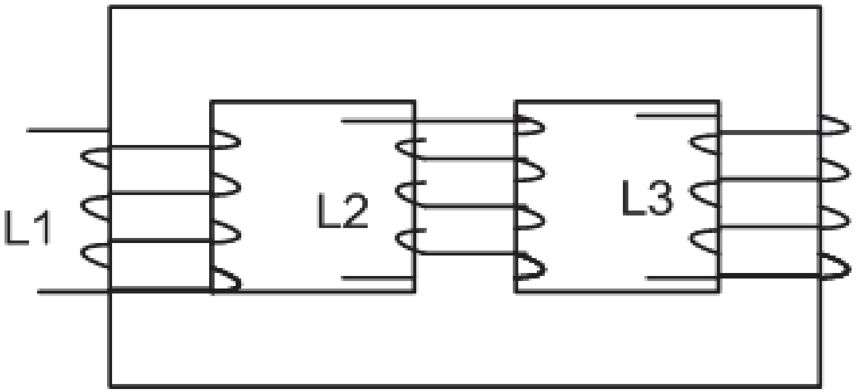
Sol: Dotted Circuit



Dotted Eqvt

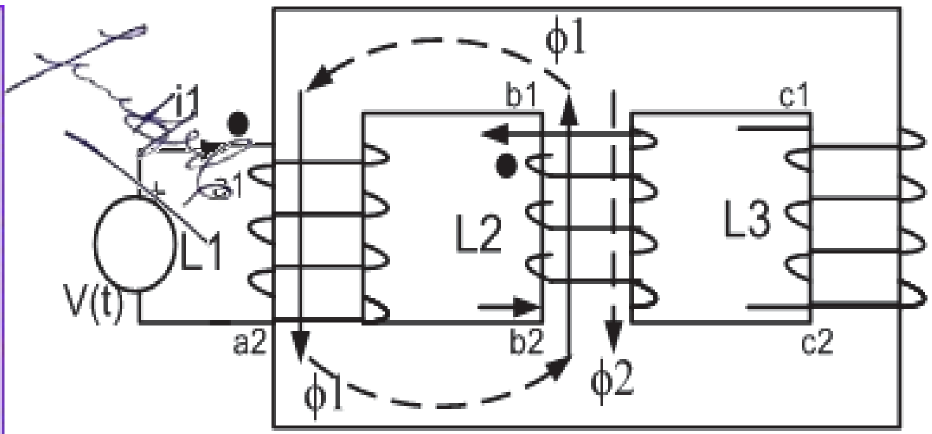


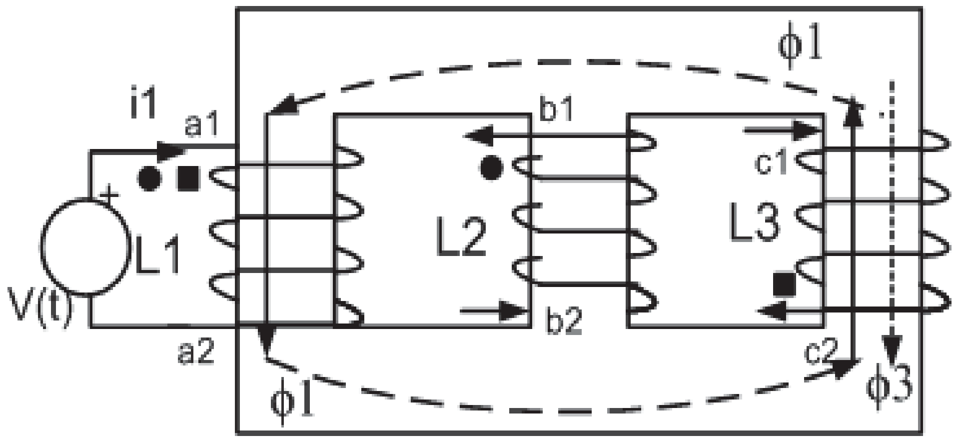
Ex. Mark the dots on the inductive coils of the following transformer

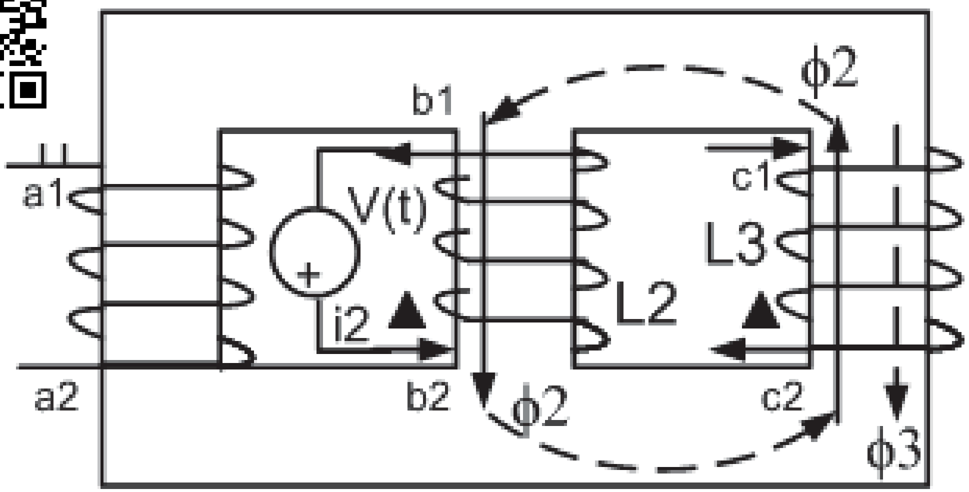


Sol: Consider that a voltage source v(t) is connected at L1 between a1 and a2, flux $\phi 1$ reaching to L2. Lenz's reaction is formation of opposite flux $\phi 2$.

The same flux $\phi 1$ will reach to L3 , And the Lenz's reaction is formation of opposite flux $\phi 3$.

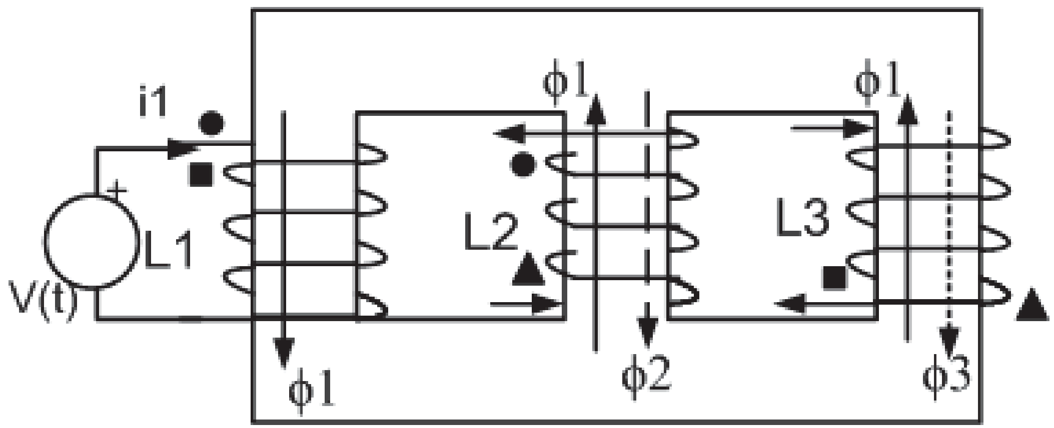






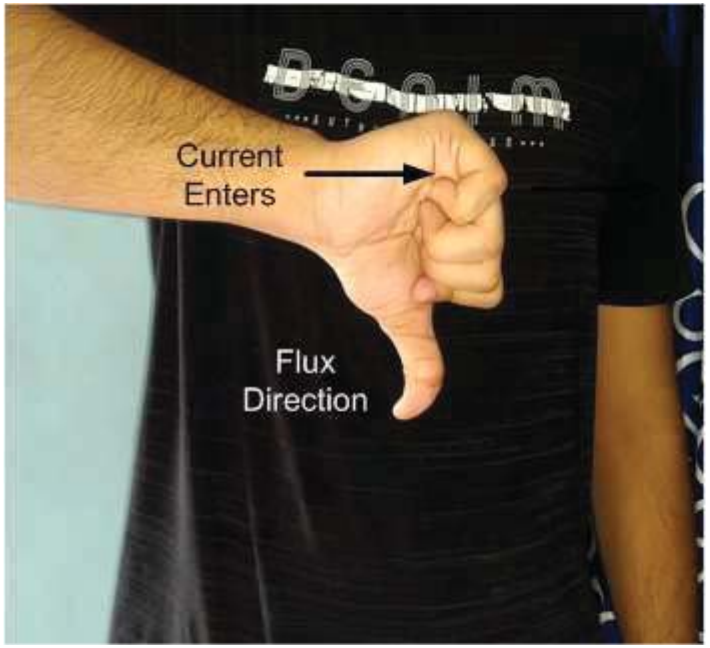
We consider here that a voltage source v(t) is connected to L2 between b1 and b2. We take the polarity suitable to previous current direction.
Thus the relation of L2 and L3 is decided.

And, here is the total dot marking .

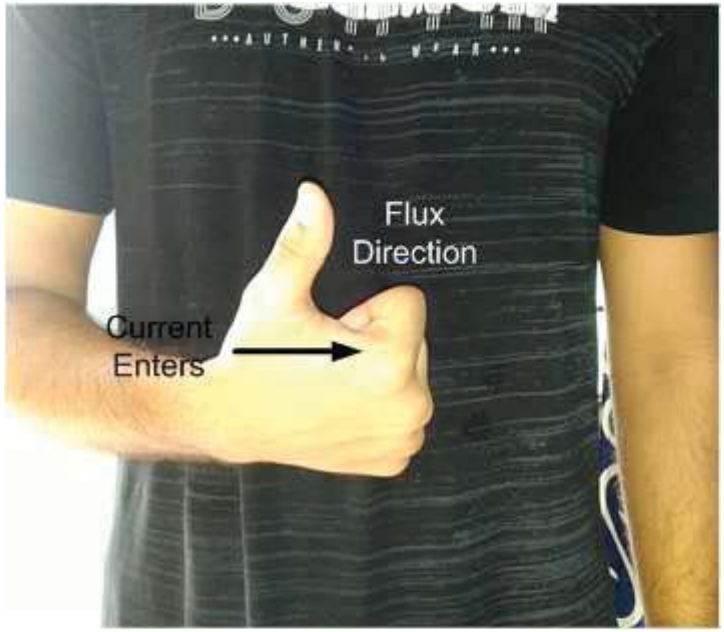


Positions of Your Hand and Fingers While deciding the Direction of Magnetic Flux

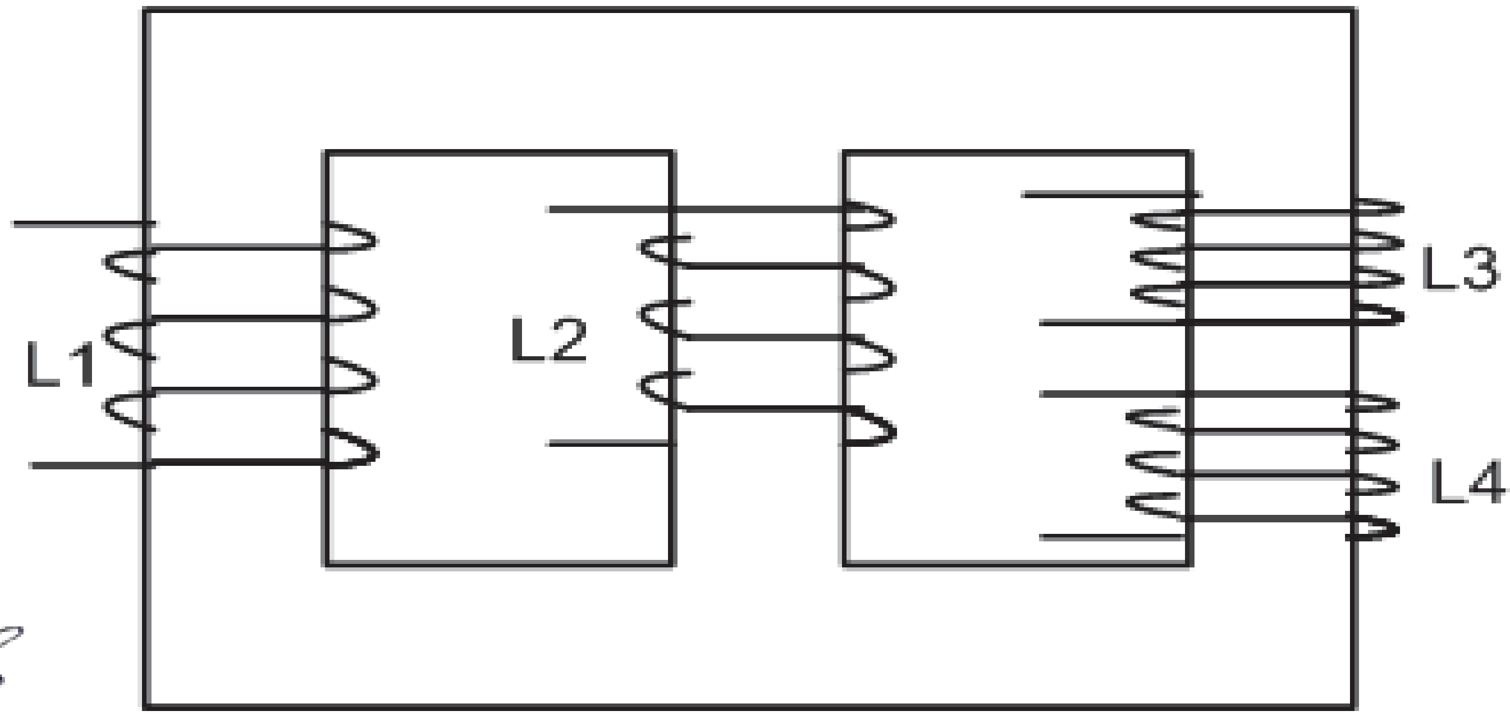
First Turn of Coil Goes Under the Core



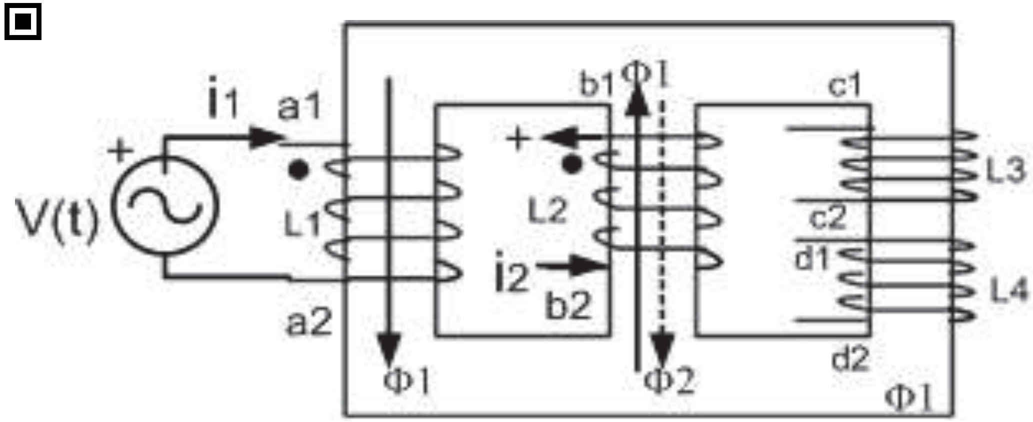
First Turn of Coil Goes Above the Core



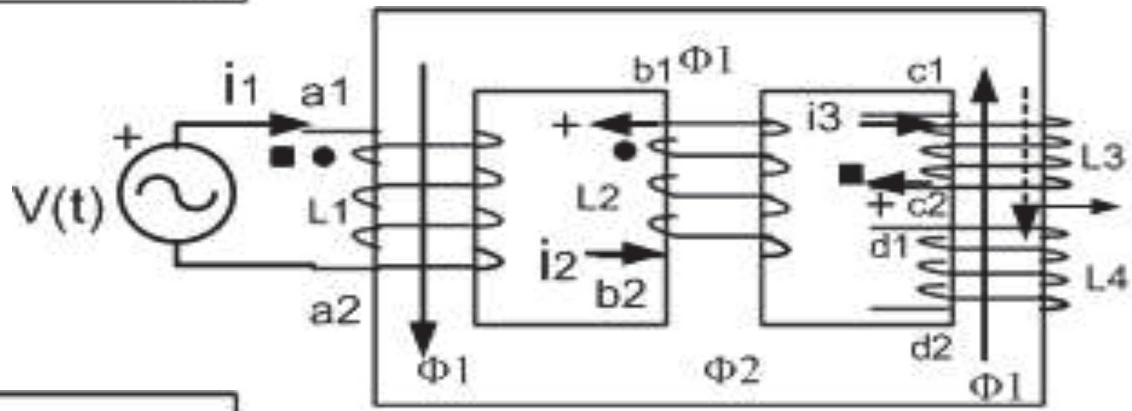
Ex. Give the dots on the following network.



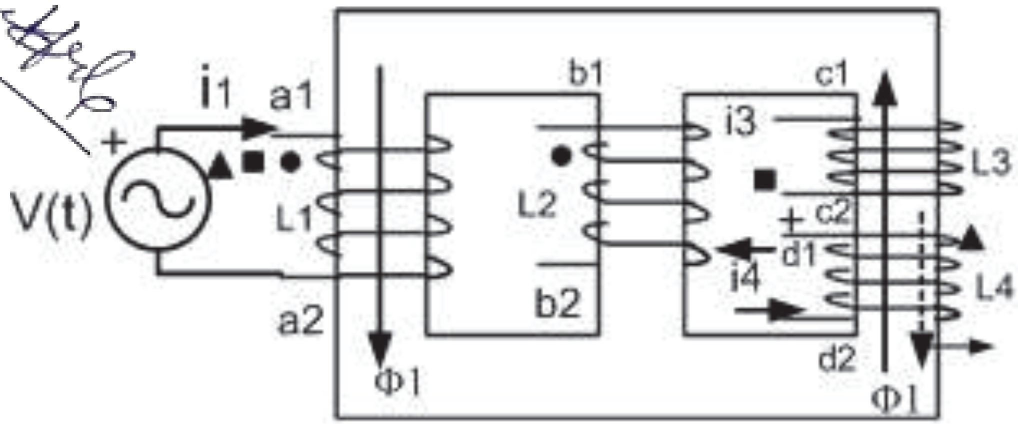
❖Influence of i1 of L1 on L2



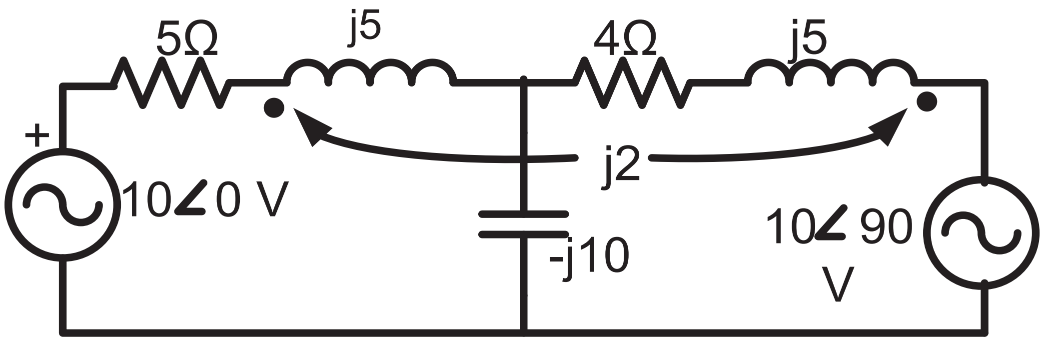
❖Influence of i1 of L1 on L3



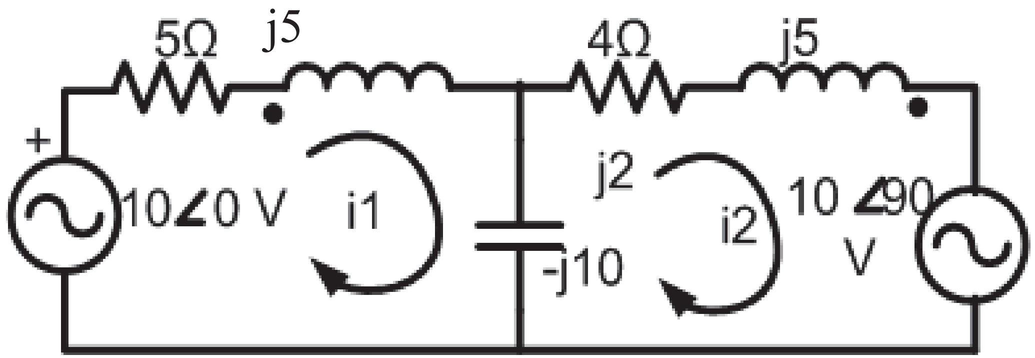
❖Influence of i1 of L1 on L4



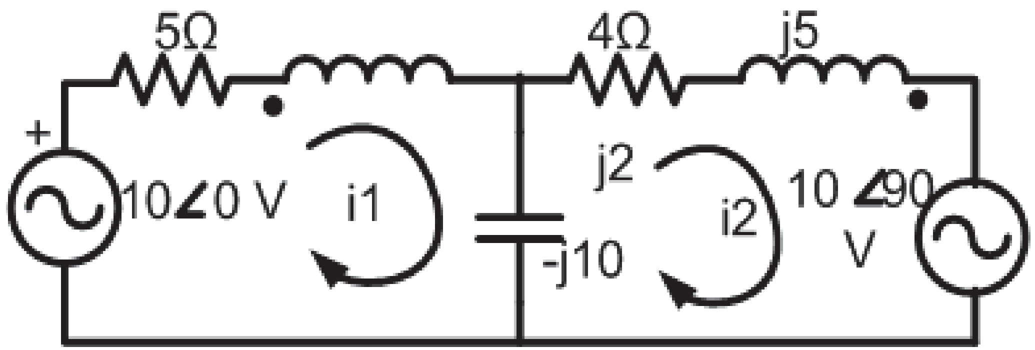
x. Write the equilibrium equation for following ckt.



Sol.



1. Select the meshes and mesh currents with a specific direction.
2. Apply KVL with all the rule.
3. If an inductor is shown with coupling (dot mark), write its all voltages in a square bracket. One essentially +ve self induced emf and other +ve or –ve mutually induced emfs .
4. When writing mutually induced emfs be careful to note whether **the current in the first coil (Coil of the mesh for which we are writing equation) is entering or leaving the dotted or undotted terminal. If the current of second coil (coil of other mesh) is doing same then take this voltage +ve. Otherwise take it –ve.**
5. **There may be more than one current passing through the other coil, lump them up (like, i1 ± i2) or write them one by one.**



Equilibrium eq for 1st mesh:

$5\,i1 + [\,j5\,i1 - j2\,i2] + (-j10)(\,i1 - i2) = 10\angle 0$

Simplified, $(5 + j5 - j10)\,i1 + (-j2 + j10)\,i2 = 10\angle 0$

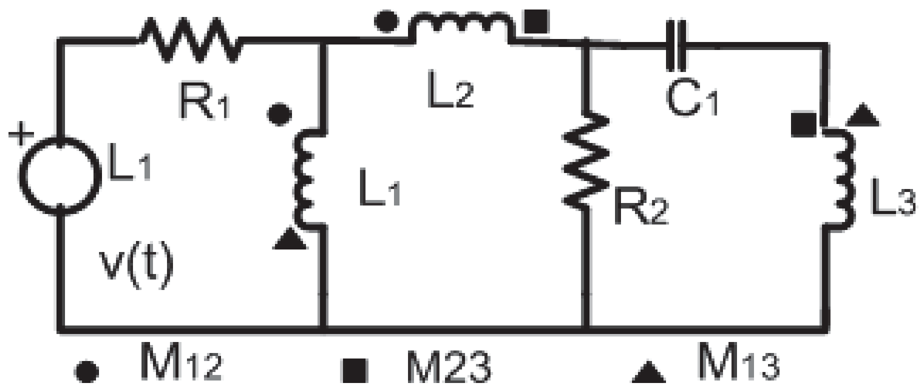
Equilibrium eq for 2nd mesh:

$-j10\,(\,i2 - i1) + 4\,i2 + [\,j5\,i2 - j2\,i1] = 10\angle 90$

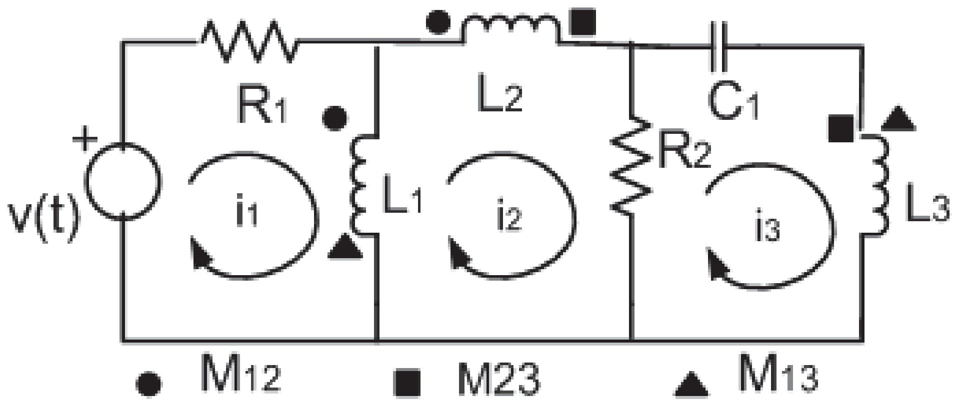
$(\,j10 - j2\,)\,i1 + (\,-j10 + 4 + j5\,)\,i2 = 10\angle 90$

$$\begin{bmatrix} (5 + j5 - j10) & (-j2 + j10) \\ (j10 - j2) & (-j10 + 4 + j5) \end{bmatrix} \begin{bmatrix} i1 \\ i2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 10\angle 90 \end{bmatrix}$$

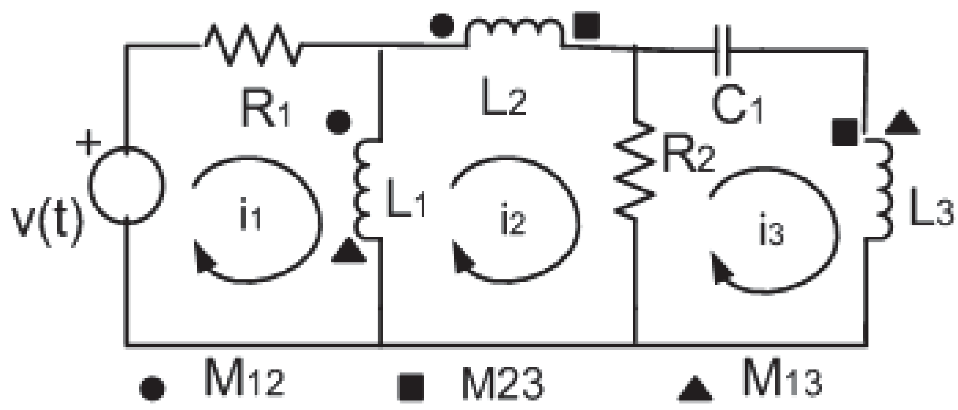
x. Write the mesh equation for the network shown



Sol:



1. Three meshes are identified.
2. Mesh Current i1, i2 & i3 are assumed clockwise.
3. Equilibrium eqs written for each mesh using KVL .



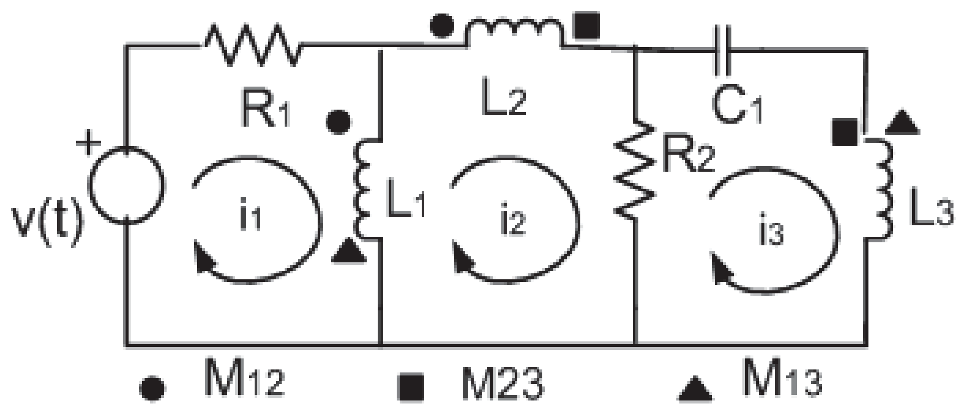
For mesh-1,

1. Voltage across L1:
   a. Self induced emf **: L1p (i1 – i2**)….two currents are passing through L1, i1 > i2.
   b. Mutually induced emf on L1 due to i2 passing through L2 : + **M12 p i2**…. Both currents, i1 through L1 & i2 through L2 are entering the dotted terminal, hence emf + ve
   c. Mutually induced emf on L1due to i3 passing through L3 : **-M13 p i2**…. the currents, i1 through L1 leaves dotted terminal but i2 through L2 is entering the dotted terminal, hence emf - ve.
2. So the equation,

$$R_1 i_1 + [L_1 p(i_1 - i_2) + M_{12} p\, i_2 - M_{13} p\, i_3] = v(t)$$

After simplification

$$(R_1 + L_1 p) i_1 + (-L_1 p + M_{12} p) i_2 - M_{13} p\, i_3] = v(t) \qquad \text{..........(1)}$$



For mesh-1,

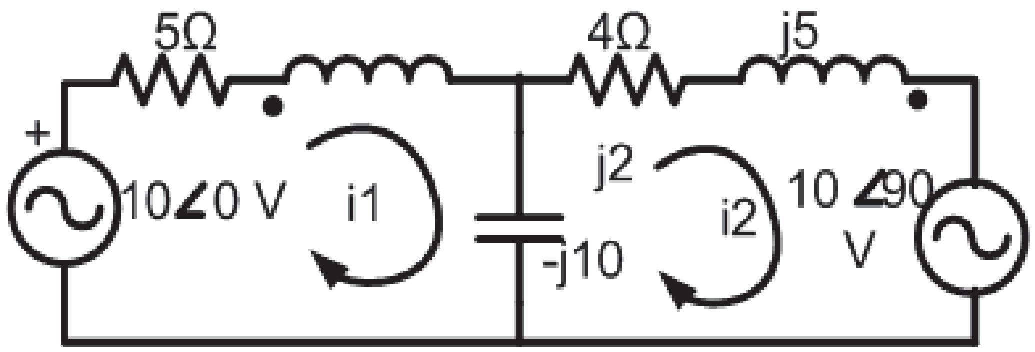
1. Voltage across L1:

   a. Self induced emf **: L1p (i2 – i1**)….two currents are passing through L1, here i2 > i1.

   b. Mutually induced emf on L1 due to i2 passing through L2 : + **M12 p i2**…. Both currents, i1 through L1 & i2 through L2 are entering the dotted terminal, hence emf + ve

   c. Mutually induced emf on L1due to i3 passing through L3 : **-M13 p i2**…. the currents, i1 through L1 leaves dotted terminal but i2 through L2 is entering the dotted terminal, hence emf - ve.

2. So the equation,



Equilibrium eq for 1st mesh:

$5\,i1 + [\,j5\,i1 - j2\,i2] + (-j10)(\,i1 - i2) = 10\angle 0$

Simplified, $(5 + j5 - j10)\,i1 + (-j2 + j10)\,i2 = 10\angle 0$

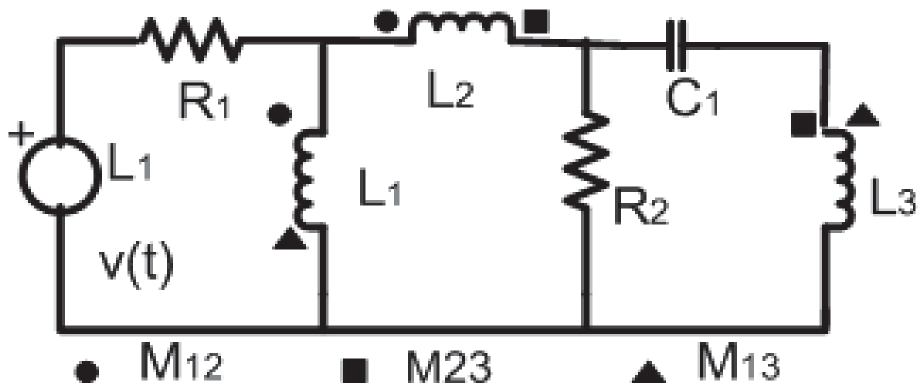
Equilibrium eq for 2nd mesh:

$-j10\,(\,i2 - i1) + 4\,i2 + [\,j5\,i2 - j2\,i1] = 10\angle 90$

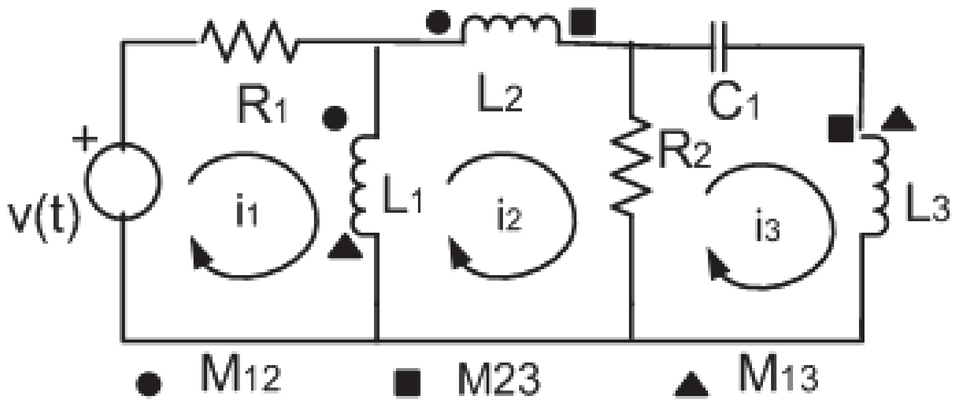
$(\,j10 - j2\,)\,i1 + (\,-j10 + 4 + j5\,)\,i2 = 10\angle 90$

$$\begin{bmatrix} (5 + j5 - j10) & (-j2 + j10) \\ (j10 - j2) & (-j10 + 4 + j5) \end{bmatrix} \begin{bmatrix} i1 \\ i2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 10\angle 90 \end{bmatrix}$$

x. Write the mesh equation for the network shown



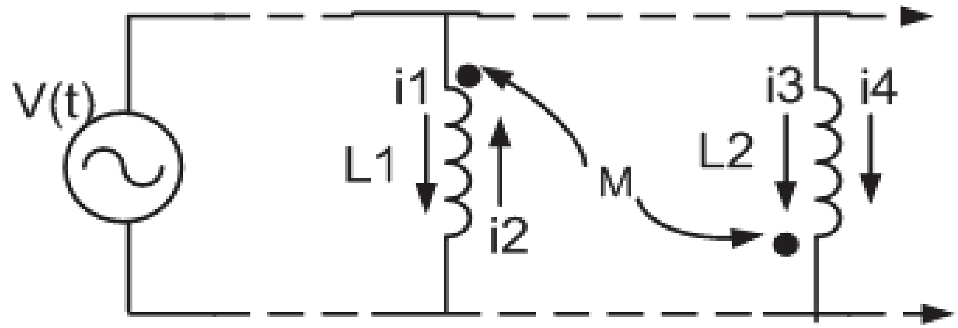
Sol:



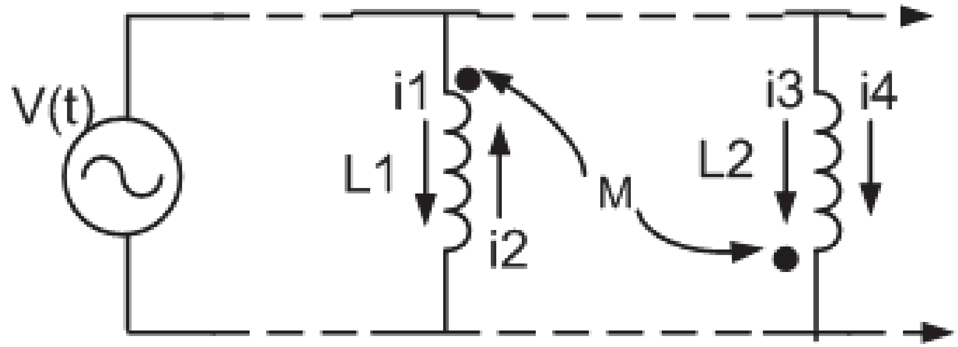
1. Three meshes are identified.
2. Mesh Current i1, i2 & i3 are assumed clockwise.
3. Equilibrium eqs written for each mesh using KVL .

rocedure to follow :

Sample Network →



1. If an inductor is shown with coupling (dot mark), write its all voltages in a square bracket.
   One essentially +ve, self induced emf and others +ve or –ve mutually induced emfs .
2. **Self Induced EMFs :**
   Inductor carries current, one or more than one, may be in same direction (added) or different directions (subtracted). One current will be greater than another, depending on, for which mesh the eq is written. Voltage due to this self induced emf will be always + ve.
3. **Mutually Induced EMFs:**
   There are two coils to compare the direction of their currents with respect to dots.
   The coil which is a member of a mesh for which we are writing eq, is the **FIRST coil (e.g.L1)**
   The another coil, going to be compared is the **SECOND coil (e.g.L2)** .
   The first coil may carry one or more than one current. But we shall consider only the mesh current. Another current will be taken into account when we go to mesh of that current.



4. Eg. L1 is **first coil** of above circuit, carries two currents i1 & i2. But only i1 will be taken to be compared. i2 will be taken when we shall be in that mesh.

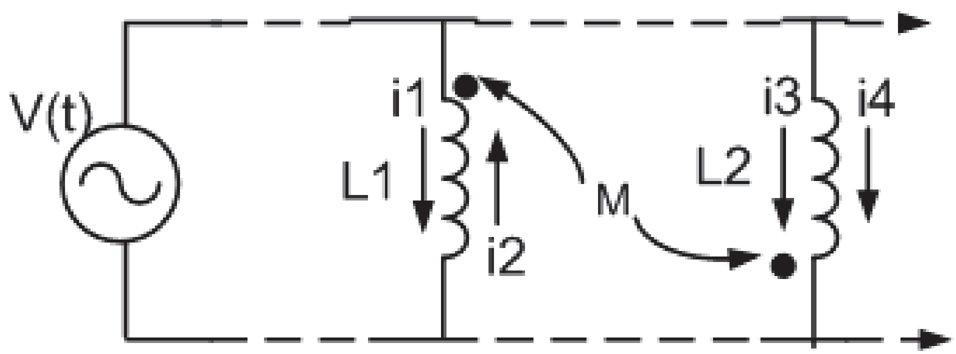
5. If L2 is the **second coil**, it may also carry two currents e.g. i3 & i4, and both are to be taken to compare. They may be in same direction (added) or different directions (subtracted).

   They may be lumped (combined) to form (i3 + i4) or (i3 – i4).

   Here they are in same direction hence added, (i3 + i4) and their direction is downward.

   If they are not in same direction, and if i3 > i4 then subtracted i.e. (i3 – i4). It's direction is the direction of i3.

6. Thus, first coil will carry only one current whereas second coil may carry any number of currents.



7. Now, the current through L1, i.e. i1 will be checked, whether entering through dotted end. It is entering the dotted end. Then check the direction of (i3+i4).

Here (i3 + i4) is not entering but leaving the dotted end. Hence this emf is taken - ve.

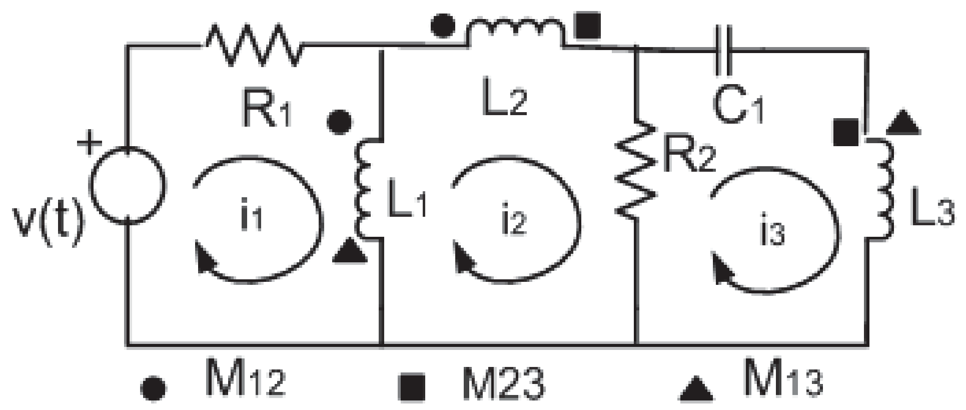
I.e. - Mp (i3 + i4).

8. So, If currents of first coil and second coil are both entering or leaving simultaneously, then the emf will be taken + ve.

But if one is entering and another is leaving or vice-versa then emf is –ve.

7. Sometimes L1 & L2 both belongs to the same mesh, but there also the same above rule will be applied in that exact way as explained.

x continued..



For mesh-1,

1. Voltage across L1:

   a. **Self induced emf : L1p (i1 – i2)**….two currents are passing through L1, i1 > i2.

   b. **Mutually induced emf :** On L1 due to i2 passing through L2 : + **M12 p i2**…. Both currents, i1 through L1 & i2 through L2 are entering the dotted terminal, hence emf + ve.

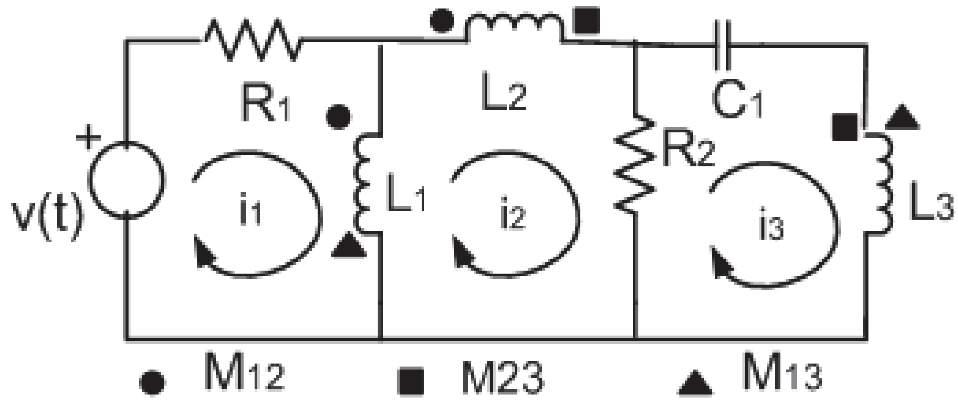
   c. Mutually induced emf on L1due to i3 passing through L3 : **-M13 p i2**…. the currents, i1 through L1 leaves dotted terminal but i2 through L2 is entering the dotted terminal, hence emf - ve. I.e. **–M13 p i3.**

2. So the equation,

$$R_1 i_1 + [L_1 p(i_1 - i_2) + M_{12} p\, i_2 - M_{13} p\, i_3] = v(t)$$

After simplification

$$(R_1 + L_1 p) i_1 + (-L_1 p + M_{12} p) i_2 - M_{13} p\, i_3] = v(t) \quad \text{..........(1)}$$



For Mesh-2,

1. Voltage across L2:

   a. Self induced emf **: L2p i2.** one currents i2 is passing through L2.

   b. Mutually induced emf on L2 due to lumped current (i2 - i1) passing through L1 in direction of i2 as i2 > i1, here i2 enters the circular dot but (i2 – i1) leaves the dot, hence emf is –ve : **- M12 p i2.**

   c. Mutually induced emf on L2 due to i3 passing through L3 : **-M13 p i3**…. the currents, i2 through L2 leaves square dotted terminal but i3 through L3 is entering thesquare dotted terminal, hence emf - ve.

2. Voltage through R2 : R2 i2.

   So the equation,

or Mesh 2:

$$L_1 p(i_2 - i_1) - M_{12}pi_2 + M_{13}pi_3] + [L_2p(i_2) - M_{12}p(i_2 - i_1) - M_{23}pi_3] + R_2(i_2 - i_3) = 0$$

Simplifying,

$$(-L_1p + M_{12}p)\, i_1 + (L_1p + L_2p - 2M_{12}p + R_2)i_2 + (M_{13}p - M_{23}p - R_2)i_3 = 0 \quad \text{----(2)}$$

For Mesh 3:

$$R_2(i_3 - i_2) + \frac{1}{C_1p} i_3 + [L_3pi_3 + M_{13}p(i_2 - i_1) - M_{23}pi_2] = 0$$

Simplifying,

$$-M_{13}pi_1 + (-R_2 + M_{13}p + M_{23}p)i_2 + \left(R_2 + \frac{1}{C_1p} + L_3p\right)i_3 = 0 \quad \text{-------(3)}$$

In Matrix form :

$$\begin{bmatrix} (R_1 + L_1p) & (-L_1p + M_{12}p) & -M_{13}p \\ (-L_1p + M_{12}p) & (L_1p + L_2p - 2M_{12}p + R_2) & (M_{13}p - M_{23}p - R_2) \\ -M_{13}p & (M_{13}p - M_{23}p - R_2) & \left(R_2 + \frac{1}{C_1p} + L_3p\right) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ 0 \\ 0 \end{bmatrix}$$

Ex. Write the Mesh basis equilibrium

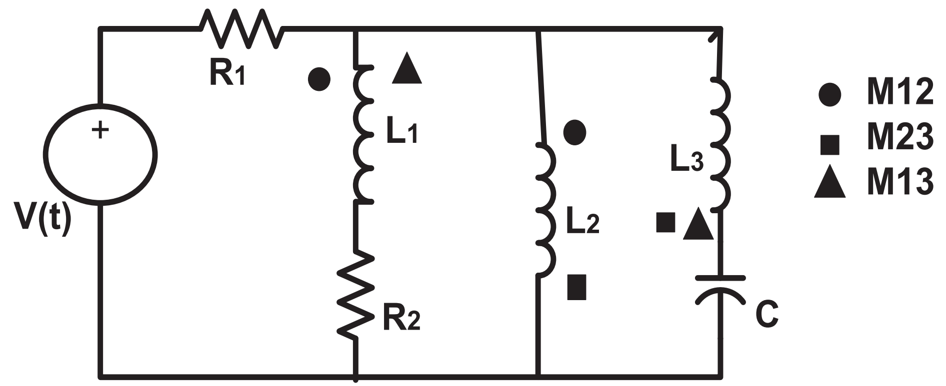
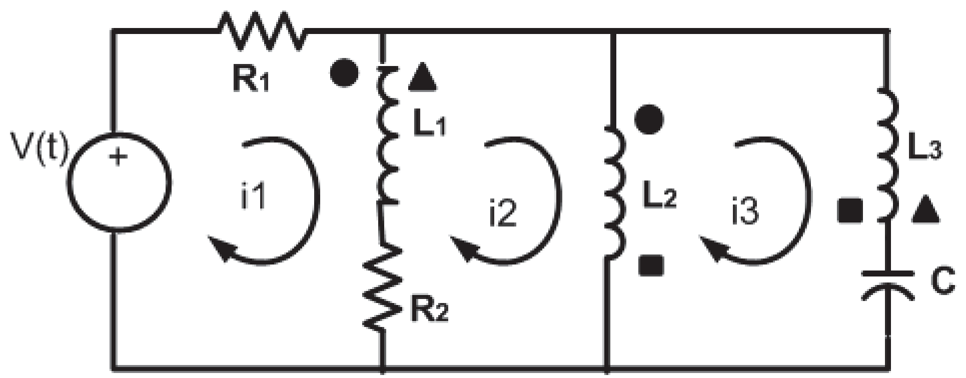
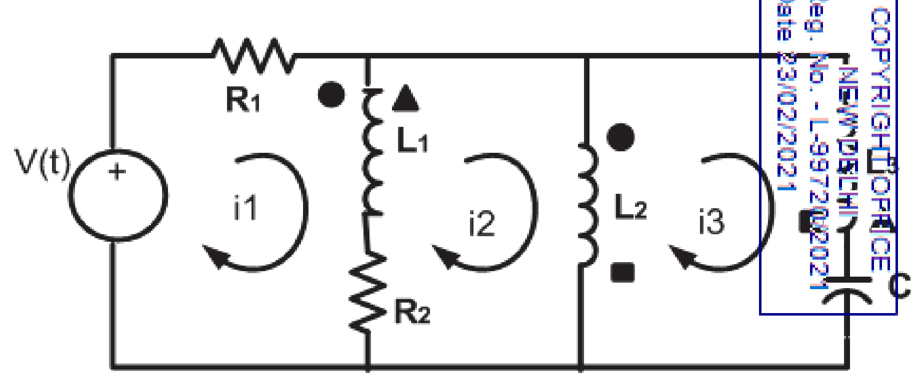


Fig 1-b

Sol:





Applying KVL in **Mesh – 1**:

1. Voltage across R1 → R1 i1

2. Voltage across L1 → (a) Self Induced emf = L1 p (i1- i2)

   (b) Mutually induced emf due to (i2 – i3) of L2,

   First current: i1(**downward**), Second current : (i2 – i3) **downward** current through L2

   Both currents entering the circular dotted ends. Hence effect +ve.

   = +M12 p (i2 – i3)

   (c) Mutually induced emf due to (i3) passing through L3, First current: i1,

   Second current : i3 **downward** current through L3.

   i1 **entering** the triangular dot but i3 is **leaving** the same of L3. Hence effect –ve.

   = - M13 p i3

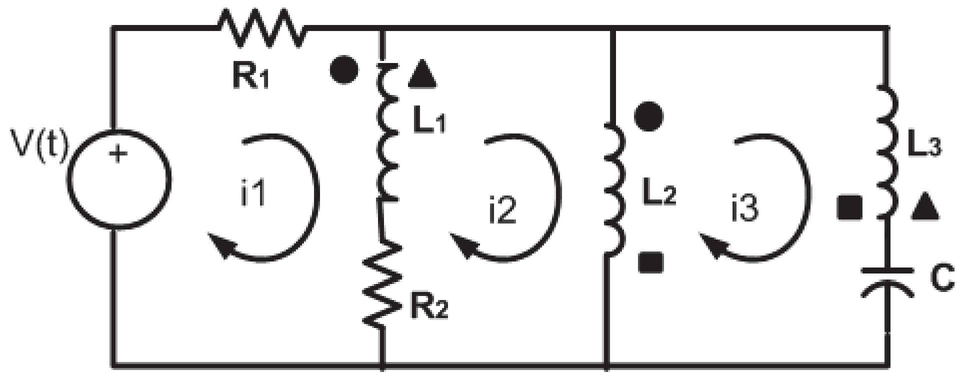
   Hence Total voltage on L1 : [L1 p (i1- i2) +M12 p (i2 – i3) - M13 p i3]

3. Voltage on R2 → R2 (i1 – i2)

So, the eql eq is: R1 i1 + [L1 p (i1- i2) +M12 p (i2 – i3) - M13 p i3] + R2 (i1 – i2)

Simplified Eq. : (R1+L1p+R2) i1 + (-L1p +M12p-R2) i2 + (-M12p – M13p) i3=v(t)

………(1)



For **Mesh-2**:

1. Voltage across L1 → (a) Self Induced emf =L1 p (i2- i1)

(b) Mutually induced emf due to (i2 – i3) of L2, First current: i2, moving **upward,**

Second current : (i2 – i3) moving **downward** through L2 .

i2 is **leaving** the circular dotted ends but (i2 – i3) is **entering**. Hence effect -ve.

= - M12 p (i2 – i3)

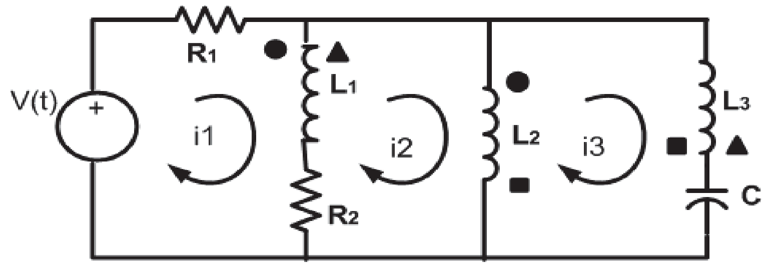
(c) Mutually induced emf due to i3 of L3, First current: i2 is **upward**,

Second current : i3 **downward** current through L3. i2 **leaving** the triangular dot, also

i3 is **leaving** the same of L3. Hence effect +ve.

= + M13 p i3

Hence Total voltage on L1 : [L1 p (i2- i1) - M12 p (i2 – i3) + M13 p i3]



2. Voltage across L2 → (a) Self Induced emf =L2 p (i2- i3)

(b) Mutually induced emf due to (i2 – i1) of L1, First current: i2, moving **downward,**

Second current : (i2 – i1) moving **upward** through L1 .

i2 is **entering** the circular dotted ends but (i2 – i1) is **leaving**. Hence effect -ve.

= - M12 p (i2 – i1)

(c) Mutually induced emf due to i3 of L3, First current: i2, going **downward.**

Second current : i3 **downward** current through L3. i2 **leaving** the square dot, also

i3 is **leaving** the same of L3. Hence effect +ve.

= + M23 p i3

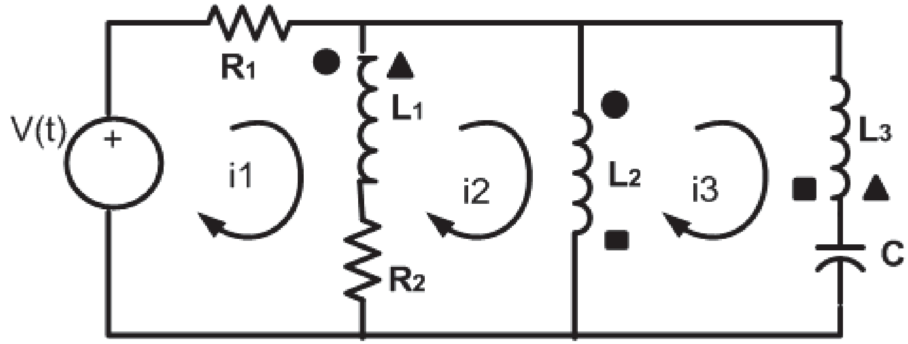
Hence Total voltage on L2 : [L2 p (i2- i3) - M12 p (i2 – i1) + M23 p i3]

So, the eql eq : R2 (i2 – i1) +[L1 p (i2- i1) - M12 p (i2 – i3) + M13 p i3]

+ [L2 p (i2- i3) - M12 p (i2 – i1) + M23 p i3] = 0

Simplified Eq : (-R2-L1p+M12p) i1 + (R2+L1p+ L2p-2M12p) i2 +

(-L2p+ M12p+M13p+M23p) i3 = 0 ………..(2)



For **Mesh-3**

1. **Voltage across L2** → (a) Self Induced emf =L2 p (i3- i2)

(b) Mutually induced emf due to (i2 – i1) of L1, First current: i3, moving **upward**,

Second current : (i2 – i1) moving **upward** through L2 .

i3 is **leaving** the circular dotted ends , also (i2 – i1) is **leaving**. Hence effect +ve.

= + M12 p (i2 – i1)

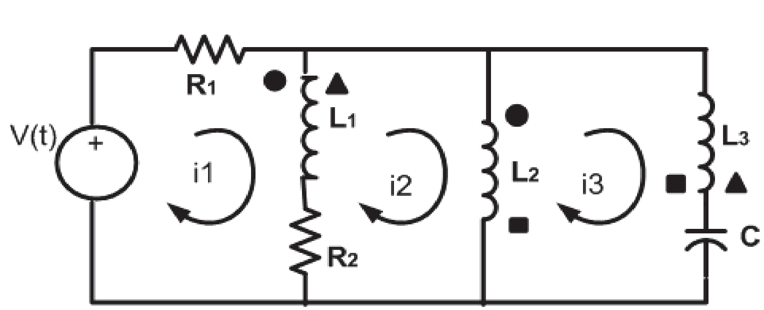
(c) Mutually induced emf due to i3 of L3, First current: i3 (of L2), going **upward**.

Second current : i3 **downward** current through L3. i3 **entering** the square dot, but

i3 is **leaving** the same of L3. Hence effect - ve.

= - M23 p i3

Hence Total voltage on L2 : [L2 p (i3- i2) + M12 p (i2 – i1) - M23 p i3]



1. **Voltage across L3** → (a) Self Induced emf =L3 p i3

(b) Mutually induced emf due to (i2 – i1) of L1, First current: i3, moving **downward**, Second current : (i2 – i1) moving **upward** through L2 . i3 **leaves** the triangular dotted end of L3, also (i2 – i1) is **leaving** at L1. Hence effect +ve.

= + M13 p (i2 – i1)

(c) Mutually induced emf due to (i3-i2) of L2, First current: i3 (of L3), going **downward.**

Second current : (i3-i2) **upward** current through L2. **i3 leaving** the square dot, but

(i3-i2) is **entering** the same of L2. Hence effect - ve.

= - M23 p (i3-i2)

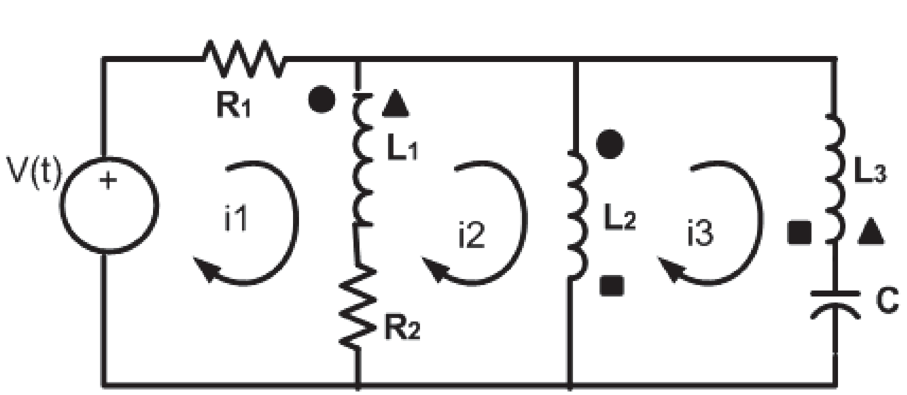
Hence Total voltage on L3 : [L3 p i3 + M13 p (i2 – i1) - M23 p i3]

3. Voltage across C → (1/Cp) i3

So, the eql eq : [L2 p (i3- i2) + M12 p (i2 – i1) - M23 p i3]

+ [L3 p i3 + M13 p (i2 – i1) - M23 p (i3-i2)] + (1/Cp) i3 = 0

Simplified Eq : (-M12p-M13p) i1 + (-L2p+M12p+M13p+M23p) i2 + (L2p+L3p – 2M23p+(1/Cp)) i3=0 **…….(3)**



(R1+L1p+R2) i1 + (-L1p +M12p-R2) i2 + (-M12p – M13p) i3=v(t) ………………..…(1)

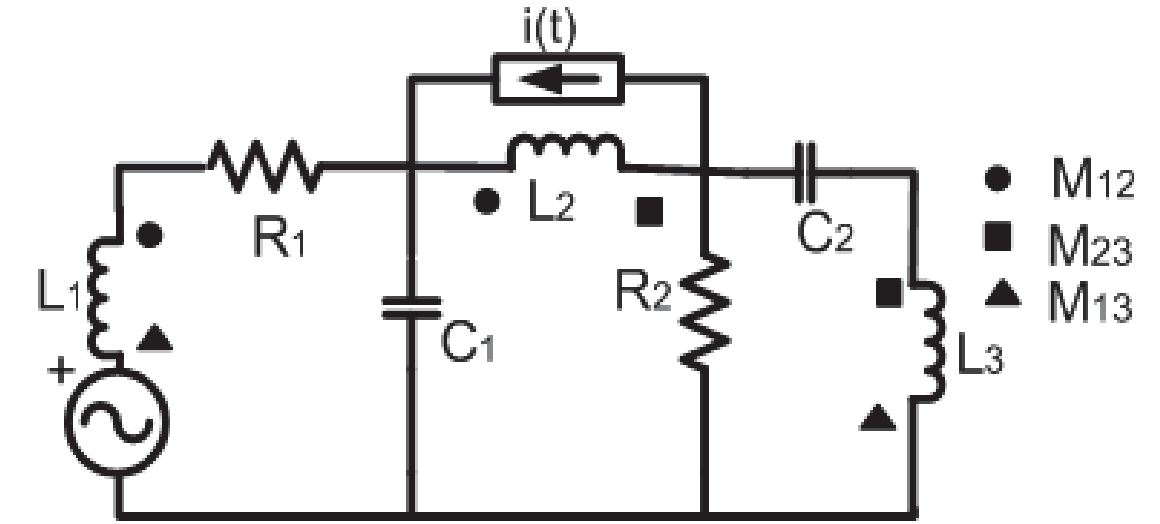
(-R2-L1p+M12p) i1 + (R2+L1p+ L2p-2M12p) i2 + (-L2p+ M12p+M13p+M23p) i3 = 0 ………..(2)

(-M12p-M13p) i1 + (-L2p+M12p+M13p+M23p) i2 + (L2p+L3p –2M23p+(1/Cp)) i3 = 0 …….(3)

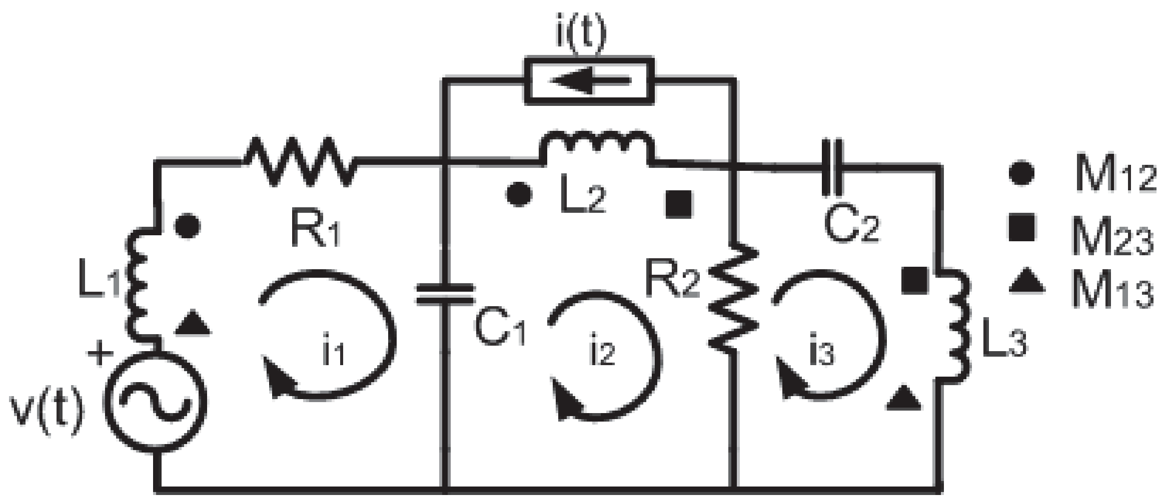
So, in Matrix form:

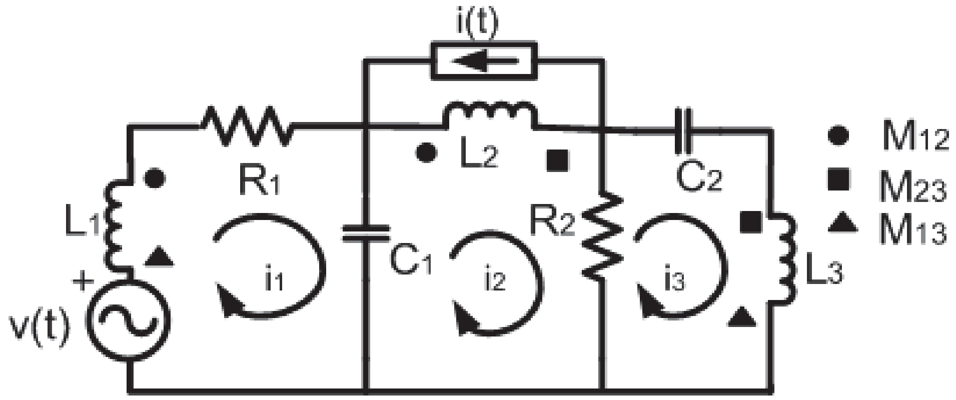
$$\begin{bmatrix} (R_1+L_1p+R_2) & (-L_1p+M_{12}p-R_2) & -M_{12}p-M_{13}p \\ (-R_2-L_1p+M_{12}p) & (R_2+L_1p+L_2p-2M_{12}p) & (-L_2p+M_{12}p+M_{13}p+M_{23}p) \\ -M_{12}p-M_{13}p & (-L_2p+M_{12}p+M_{13}p+M_{23}p) & \left(L_2p+L_3p\ -2M_{23}p+\frac{1}{C\,p}\right) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ 0 \\ 0 \end{bmatrix}$$

Ex. Write the mesh equation for the network shown.



Sol.





For Mesh-1

[L1p i1-M12p (i2+i(t)) –M13pi3] +R1i1+ (1/C1p) (i1-i2) = v(t)

Simplified Eq,

(L1p+ R1+(i/C1p)) i1 +(-M12p- (1/C1p)) i2 + (-M13p) i3 = v(t) + M12p i(t)….(1)

For Mesh-2:

(1/C1p) (i2-i1) +[L2p (i2+i(t)) –M12p i1 - M23pi3] +R2(i2-i3) = 0

Simplified,

(- (1/C1p) –M12p)i1 + ((1/C1p) +L2p+R2)i2 + (- M23p-R2)i3 = -L2pi(t)……………..(2)

For Mesh-3:

R2 (i3-i2) + (1/C2p) i3 +[L3p i3 – M13p i1 –M23p(i2+i(t))] =0

Simplified,

(-M13p)i1 + (-R2 – M23p) i2 + (R3+ (1/C2p) +L3p) = +M23pi(t)……………….(3)